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THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

Edited by

GEORGE E. HALE
Mount Wilson Observatory of the Carnegie Institution of Washington

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SEPTEMBER 1931

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PHOTOGRAPHS OF THE MILKY WAY IN CYGNUS AND CEPHEUS

By FRANK E. ROSS

ABSTRACT

Reproductions of two negatives of the Milky Way in Cygnus and Cepheus, each covering a wide field, and of a negative of the veiled nebulae in Cygnus, taken on a larger scale, are given. Various problems connected with photographing the sky with wide-angle lenses are considered. The characteristics of the lens are pointed out, and a simple method of curving the photographic plate, in order to improve the field, is given. A very simple and efficient backing for the plates is described. The question of the proper developer to bring out the faint extended nebulae on the negative is taken up. Returning to the reproductions, an error sometimes made in judging relative magnitudes of nebulae or stars from prints or reproductions is pointed out, and application made to stellar photometry.

Plates V and VI are reproductions of negatives taken by the writer at Flagstaff on October 13 and 14, 1930, respectively, with a wide-angle lens of the author's design. Its aperture is 5 inches (12.7 cm), and focal length 35 inches (88.9 cm), and is mounted in a stainless steel cell. It was made by J. W. Fecker & Company. The camera and plate-holder are of duraluminum. The camera was attached to the mounting of the 15-inch reflector of the Lowell Observatory. The plates used, Imperial Eclipse soft, blue sensitive, were 14 inches square. On account of defects in backing, however, the reproductions do not fully include the field covered. The exposure time was three hours on each field. Plate V is centered on γ Cygni, and includes the region between $19^h36^m - 21^h12^m$ in right ascension and $29°–50°$ in declination. Plate VI is centered on π_1 Cygni, including $20^h36^m - 22^h36^m$ in right ascension and $39°–59°$ in declination,

equinox of 1855. Taken together they cover the most interesting part of the northern Milky Way. Their extent is such that the mutual relationship of the many features of this part of the sky can be seen at a glance. Plate V corresponds closely in extent to the small-scale Plate 51, No. 1, of Barnard's *Atlas of Selected Regions of the Milky Way*. It is to be regretted that the plates could not be reproduced in their full size. The scale of the reproduction is 8 mm to the degree.

The veiled nebulae in Cygnus, N.G.C. 6960 and 6992, are well shown in the lower left-hand corner of Plate V, notwithstanding the fact that they are 11° from the center of the plate, and accordingly lose considerable light. This loss is largely due to the vignetting action of the lens, as a result of which some of the faint details between the two main nebulae are lost. The triangular patch of nebulosity northeast of 6960 has been the object of special study by Dr. J. C. Duncan, who has secured an exposure of it of nine hours' duration with the 100-inch reflector at Mount Wilson.[1] The nebulae 6960 and 6992 doubtless form a physical system, which includes the intervening material. There may be some question of its being a spherical shell, for there is a paucity of material southwest of 6992. In order to show this clearly I have prepared a third negative from the very excellent negative made of the region by E. E. Barnard and D. W. Morehouse on July 15, 1909, with the 10-inch Bruce telescope of the Yerkes Observatory, for which the exposure time was 343 minutes. This has been reproduced in Plate VII, and is an enlargement from the original negative of 1.5. The scale is 34 mm per degree. In order to reproduce the very faint intervening patches of nebulosity the brighter nebulae are necessarily overexposed, so that the prints are distorted photometrically. The long fine filament running south from the triangular nebulosity, suggesting a stream of water escaping from a funnel, is to be especially noted.

Plate V brings out very clearly the fact that 6960 lies on the exact edge of a very extensive irregular dark nebula. The difference in stellar density east and west of 6960 is strikingly shown on a negative of this region taken by Dr. Duncan with the 100-inch Hooker telescope.[2]

[1] *Popular Astronomy*, **37**, 580, 1929.

[2] *Astrophysical Journal*, **57**, Pl. 13, 1923.

PLATE V

MILKY WAY IN CYGNUS

Scale: 1°=8 mm.

It is the plan of the writer, using this lens and equipment, to cover at least the northern sky with long exposures. In perfecting the technique connected with securing the very best results possible, various problems have been encountered, which will be briefly outlined, as they may be of help to others.

With regard to the lens itself, there is a very slight curvature of field, amounting to about 0.7 mm at the edge of the 14-inch plate, which it was desirable to eliminate. To do this I have used a modification of the method of curving the photographic plate devised and successfully applied by Dr. V. M. Slipher for the 13-inch Lawrence Lowell telescope. The modified method is especially well adapted to a square plate. Four metal strips, about 170 mm long and 2 mm high, with top surface ground to the required circular section, are screwed to the middle of the four ledges forming the bed of the plate in the holder. Near the four corners of the back, which in the present case is hinged to the plate-holder, are four adjustable screws piercing it. When these are properly adjusted, and the back locked, after the plate has been inserted, the plate is forced to the proper curvature. On account of the very small variation in thickness of plates of the same emulsion, it is generally not necessary to change the adjustment of the screws once they are set.

Theoretically, with this particular type of lens it is not difficult to eliminate completely curvature of field, so that a curved plate would not be necessary. This would be secured, however, at the expense of a slight amount of astigmatism. The present lens is a compromise, as indeed all lenses are, with respect to their numerous aberrations. By reducing the astigmatism and leaving a small amount of curvature of field outstanding, the lens has proved to be well adapted to out-of-focus photometry. The requirements in order to produce round disks of uniform density are freedom from spherical aberration, from coma, and from astigmatism. Field curvature is not so important.

BACKING OF PLATES

A notion quite commonly held is that the backing of plates is for the sole purpose of eliminating the halation ring surrounding the bright stars. I have shown elsewhere[1] that in addition it eliminates a veiling of the whole plate in the case of long exposures, the amount

[1] *Ibid.*, **67**, 288, 1928.

of the veiling depending upon the integrated brightness of all the objects on the plate. I have also found backing very useful in making positives and second negatives for reproduction, as in the plates accompanying this article. The veiling of the unbacked plate is very noticeable.

The relative amount of veiling action of an unbacked plate depends upon the opacity of the emulsion and upon the form of its distribution-curve. At my request Mr. P. C. Keenan has determined the photographic opacities of a number of emulsions of different types. The variation is smaller than would be expected, the average value of the opacity is 10, the total range being 30 per cent. From this, and assuming that one-half of the light passing through the emulsion is returned by the unbacked glass, we see that roughly 5 per cent of the incident light is effective as veiling.

Great difficulty was experienced in securing a suitable backing formula, the principal defects found being lack of perfect optical contact, and numerous small bubbles. Finally, at the the suggestion of Dr. V. M. Slipher, a simple mixture of glucose and burnt umber was tried, which was found to be ideal.[1] At my request the index of refraction of the hardened glucose was determined by C. W. Frederick, of the Eastman Kodak Company. Its value for the D line was 1.506, with a dispersion factor, $\nu = 55$. This is very close to that of ordinary glass, so that the optical requirement is well met. The freedom from bubbles when this backing is spread over the plate is noteworthy, for some commercially backed plates are found to be quite defective in this regard.

DEVELOPMENT

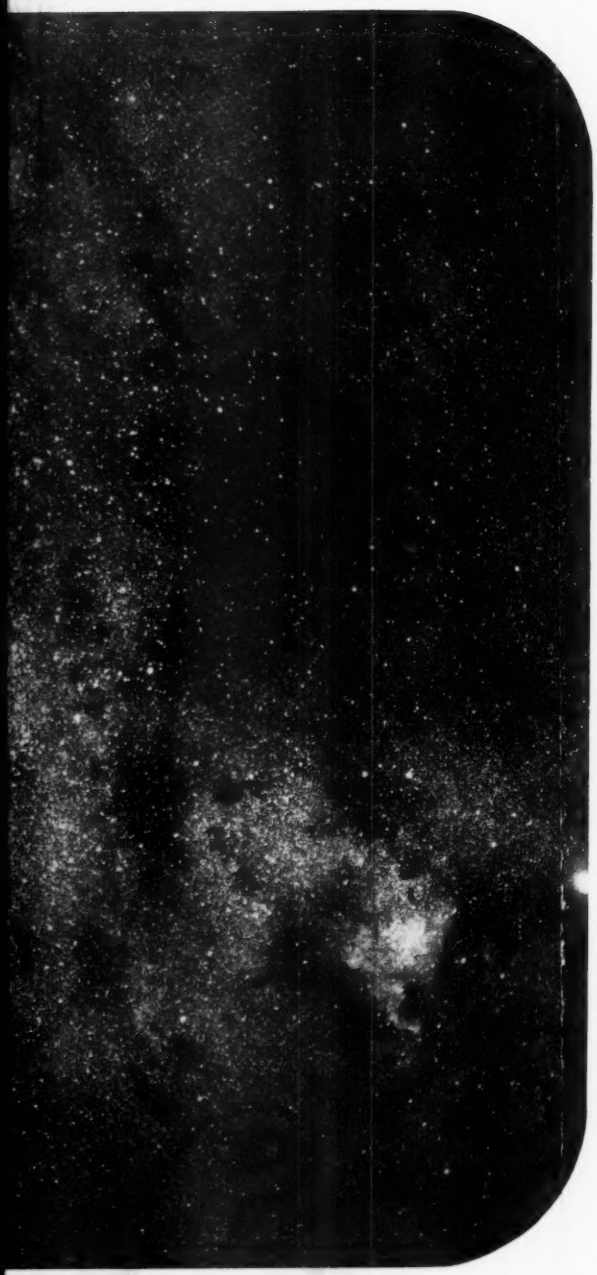
A great many experiments were made, and measurements taken, to determine the best developer and development time, for the faint nebulosities. It is to be pointed out that the requirements are quite different from those holding in spectroscopy or stellar photography, for, in registering of faint nebulae, threshold densities are increasingly important as the size of the nebula increases. In stellar images

[1] Mix the commercial glucose while hot with an equal amount of hot water, adding a small amount of alcohol as a dryer. Keep as a stock solution. Mix as required with double its volume of burnt umber. Apply with a soft camels' hair brush, kept soft by suspension in water. For quick drying a drying box is recommended. Wash off before development, with plate completely immersed in water, emulsion side down.

MILKY WAY IN CYGNUS AN

Scale: 1°=8 mm

VI



S AND CEPHEUS

mm.

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and spectral lines threshold densities are not involved; in fact, the densities in those cases are probably well up on the straight portion of the characteristic curve. Now, it is a characteristic of the so-called Process developers, of which the Eastman formulae D9 and D11 are examples, to develop the threshold densities much more slowly than the higher densities. This is a property of heavily bromided developers. They are accordingly useful in stellar and spectral photography, when care is taken to keep the development time sufficiently short, in order to eliminate the threshold densities surrounding the fine images of star or spectral line, thus increasing sharpness and resolution. On the other hand, if one is concerned with the faint extended nebulae, or even the small spiral or globular nebulae which have sensible disks, development time should be greatly prolonged, when this type of developer is used. Accordingly, I have thought it best to discard developers of the Process type in work involving the faint nebulae, and have adopted a lightly bromided pure-metol developer, such as the Hauff metol formula. This has sturdy keeping qualities, an advantage when great quantities are needed, for the development of large plates. With this developer the threshold exposures keep pace in development with the heavier exposures. At my request Mr. Keenan has carried out a series of tests, in particular, a comparison of D11 with the Hauff one-solution metol developer, using Imperial Eclipse plates. The following are the values of the gammas or contrasts which were found (temperature 68°):

Dev. Time.	D11	Metol
3*m*.....	0.62	1.07
6.....	1.17	1.12
12.....	1.45	1.18

The great speed of development of the metol, as compared with D11, and the higher final contrast, or gamma-infinity of D11, are noteworthy. However, the characteristic curves from which the table was compiled show that for the lower densities the development time of six minutes with metol is as efficient as the development time of twelve minutes with D11. It should be pointed out that the factor of high contrast of D11 for full development, that is, for a development time which will show the faint nebulosities, is a disadvantage

when it comes to reproduction. The developer used for the negatives from which Plates V and VI were made was a Wallace contrast developer. It appears to be unusually slow-acting on threshold exposures, so that the development time was necessarily prolonged, and the contrast was therefore very high. The metol developer undoubtedly would have been better, for it would have missed nothing in faint nebulae, and at the same time would have modulated the brighter objects, an advantage in reproduction.

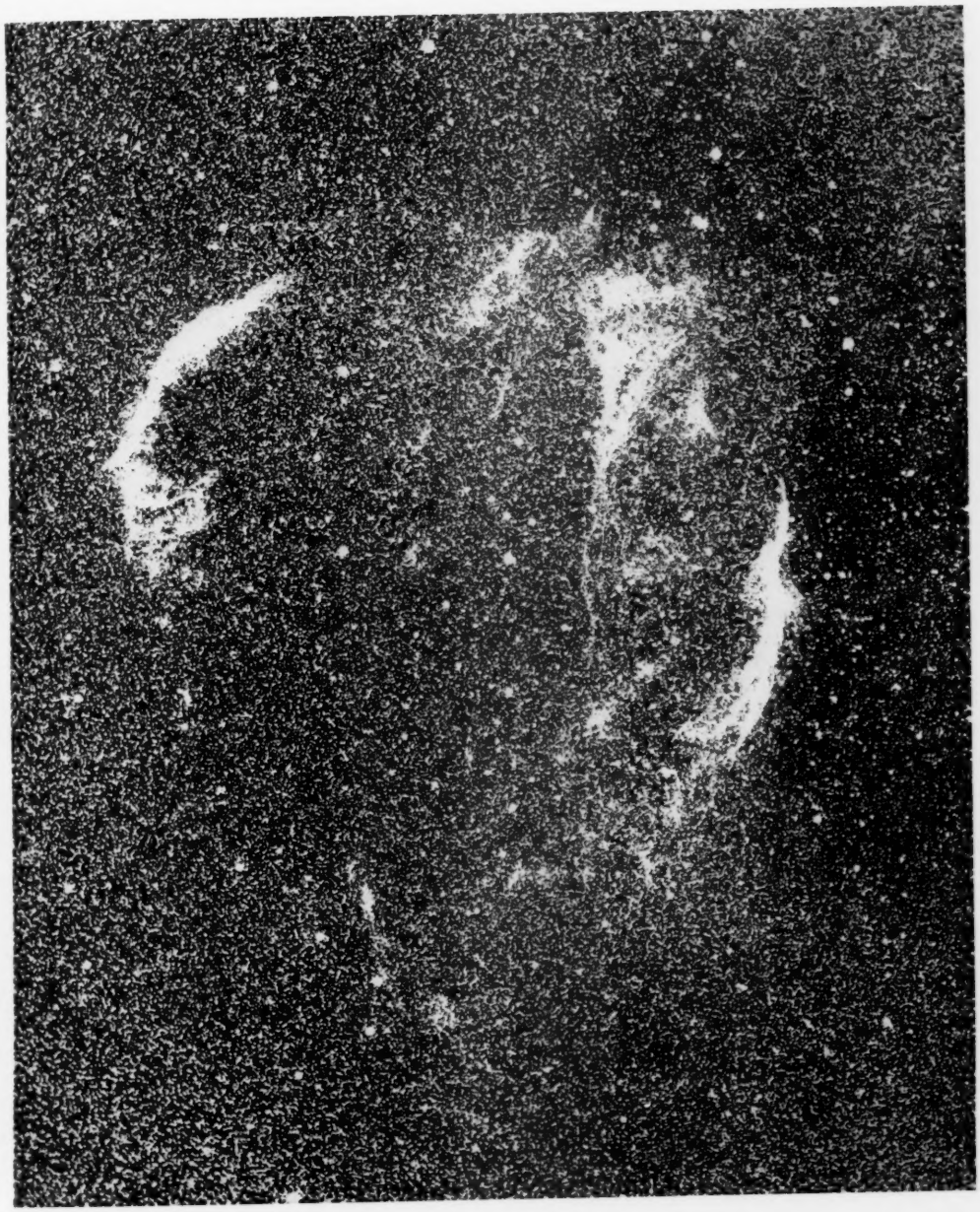
RELATIVE MAGNITUDES FROM PRINTS

It has already been noted that the relative apparent magnitudes of the extended nebulae in general cannot be correctly judged from reproductions of prints such as accompany this paper. It is to be pointed out that a similar distortion exists in the case of stellar images on these reproductions, the effect being a consequence of differences in spectral type, combined with the outstanding chromatic errors of the objective. It is well shown in Plate 44 of Barnard's *Atlas*, in the case of the star B.D. 40°4093, mag. 7.2, spectrum B8. On the original plate this star has a fuzzy outline, owing to out-of-focus violet light, which, in the processes of reproduction, becomes of a full density, so that in the resulting print the star appears much too bright. The same phenomenon is necessarily present on the original negative to a less degree; indeed, this is one of the factors entering into the color equation of an objective. It is more important on long exposures; in fact, relative apparent magnitudes can be reversed by increasing exposure time. An important corollary is that we cannot speak of the color equation of an objective without specifying the exposure time. There should be minor variations due to condition of sky. These factors, taken in conjunction with many others, make numerous pitfalls in exact focal photometry with refractors.

I am greatly indebted to Director V. M. Slipher for the opportunity to take these and other negatives at the Lowell Observatory, which is especially well located for work of this kind; and to Mr K. Newman for his assistance at the telescope.

YERKES OBSERVATORY
May 1931

PLATE VII



Veiled Nebulae in Cygnus

Scale: 1° = 34 mm.

EFFECT OF SPACE ABSORPTION ON THE CALCULATED DISTRIBUTION OF STARS[1]

By FREDERICK H. SEARES

ABSTRACT

Theoretical relations between absorption and space density.—The true distribution of the space density of stars, Δ, can be expressed in terms of the absorption function A and the fictitious distribution of density, D, obtained by ignoring the absorption. D can always be found from star counts with the aid of the luminosity function. For the case of selective absorption A can be found, theoretically at least, by deriving D from both photographic and photovisual star counts and combining the results with the general equation connecting Δ, D, and A.

Applications for special values of the absorption.—Calculations based on Mount Wilson photographic star counts and a constant coefficient corresponding to an absorption of 0.67 mag. per 1000 parsecs indicate that the true space density in the galactic plane would increase in general with increasing distance from the sun. An increase in the direction of the center of the galactic system is possible and perhaps to be expected, but scarcely in other directions. Similar calculations for other forms of A make it probable that the absorption is not constant in longitude or for any great distance in the galactic plane.

1. THEORETICAL RELATIONS

Recent determinations of coefficients of interstellar absorption which are thought to be independent of longitude and constant over great intervals of distance[2] raise again a question as to the influence of absorption on the space distribution of stars. Both Kapteyn[3] and Seeliger[4] found that, the observed distribution of stellar magnitudes being what it is, a general absorption of more than 0.001 or 0.002 mag. per parsec would imply an increase in space density of stars with increasing distance from the sun; and investigations by Halm[5] led to a similar conclusion.[6]

[1] *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 428.

[2] Trumpler, *Lick Observatory Bulletins*, **14**, 154 (No. 420), 1930; *Publications of the Astronomical Society of the Pacific*, **42**, 214, 267, 1930; van de Kamp, *Astronomical Journal*, **40**, 145 (No. 945), 1930.

[3] *Astronomical Journal*, **24**, 115 (No. 566), 1904.

[4] *Abhandlungen der K. Bayerischen Akademie der Wissenschaften*, **25**, Abh. 3, p. 47, 1909.

[5] *Monthly Notices of the Royal Astronomical Society*, **77**, 243, 1917; **80**, 162, 1919.

[6] For a general summary of results cf. Kienle, *Jahrbuch der Radioaktivität und Elektronik*, **20**, 1, 1923.

Kapteyn's discussion does not pretend to be more than a general indication of results; Seeliger's is obscured by a needless complexity of formulae and is, moreover, based on a density law of restricted applicability. On this account it seems worth while to make the calculation once more by an independent method and with the aid of modern data.

In the absence of absorption the fundamental equation of stellar statistics is

$$N(m)dm=\omega dm\int_0^\infty \phi(M)\Delta(\rho)\rho^2 d\rho\ , \tag{1}$$

where N and ϕ are the frequency functions for apparent and absolute magnitudes and Δ represents the distribution of space density as a function of the distance ρ. The zero-point for M is defined by

$$M=m-5\log\rho\ , \tag{2}$$

in which ρ is expressed in parsecs. The volume element $\omega\rho^2 d\rho$ of a thin spherical shell, multiplied by the density Δ, represents the total number of stars in the element. The remaining factor ϕ is the probability of the occurrence within this element of a star whose apparent magnitude lies between m and $m+dm$. Summation over all similar shells gives the total number of stars having magnitudes between these limits, which is the quantity represented by the left member of equation (1).

In case absorption is present, we have a similar expression

$$N(m_0)dm_0=\omega dm_0\int_0^\infty \phi(M)\Delta(\rho)\rho^2 d\rho \tag{3}$$

involving the observed apparent magnitude m_0, which is connected with m, the value corrected for absorption, by the relation

$$m=m_0-5A(\rho)\ , \tag{4}$$

where the absorption term A is specified only as a function of the distance, although it is recognized that both galactic latitude and longitude may be involved. We have further in (3)

$$M=m_0-5A-5\log\rho\ . \tag{5}$$

Equation (3) may be transformed by introducing the variable ρ_0 defined by

$$\log \rho_0 = A + \log \rho \qquad \text{or} \qquad \rho_0 = \rho e^{\frac{A}{\mu}}, \tag{6}$$

where μ is the modulus of logarithms to the base 10. From (5) and (6)

$$M = m_0 - 5 \log \rho_0 \tag{7}$$

and

$$\rho^2 d\rho = \frac{\rho_0^2}{F} d\rho_0, \tag{8}$$

where

$$F = e^{\frac{3A}{\mu}} \left(1 + \rho \frac{A'}{\mu}\right) \tag{9}$$

and A' is the derivative of A with respect to ρ. If, further, we write

$$D(\rho_0) = \frac{1}{F} \Delta(\rho), \tag{10}$$

equation (3) becomes

$$N(m_0) dm_0 = \omega dm_0 \int_0^\infty \phi(M_0) D(\rho_0) \rho_0^2 d\rho_0 . \tag{11}$$

If this integral equation for D can be solved, the true density distribution is at once given by (10), (9), and (6):

$$\Delta(\rho) = D\left(\rho e^{\frac{A}{\mu}}\right) e^{\frac{3A}{\mu}} \left(1 + \rho \frac{A'}{\mu}\right). \tag{12}$$

Equation (11) is not merely similar to (1) in form, but admits of a similar interpretation. Moreover, it is actually the basis of all solutions for the density function which start with the observed distribution of apparent magnitudes and ignore the existence of absorption. Disregard of absorption means the omission of the term $5A$ in equation (5), in other words, the substitution of $\log \rho$ for $A + \log \rho$; but this is equivalent to the introduction of the distance ρ_0 defined by (6). Such solutions therefore treat m_0 as though it were the apparent magnitude m of a star at the fictitious distance ρ_0, and hence within

the volume element $\omega\rho_0^2 d\rho_0$. The fundamental equation solved is therefore (11), where $D(\rho_0)$ is the density within the shell ρ_0 to $\rho_0+d\rho_0$. The result of the solution is accordingly the fictitious distribution of density expressed by $D(\rho_0)$.

Equation (12) shows, however, that the true distribution Δ may be found from D as soon as the absorption function A becomes known. This equation is general in that it makes no assumption as to the form of either D or A, other than a dependence on the distance. The influence of galactic latitude and longitude may of course be investigated by using (11) with values of N and ϕ which correspond to specific values of these co-ordinates.

The evaluation of Δ therefore turns upon the determination of D and A. Theoretically at least, equations (11) and (12) afford a complete solution, not only for Δ, but also for A itself, provided the absorption is of the selective type. Schwarzschild's general solution[1] of equations of the type of (11) is well known and assures us that D can be found. But if the absorption is selective as presupposed, the value of D will depend on whether the values used for N and ϕ are based on photographic or visual (or photovisual) data. The true density Δ is, however, independent of this choice and must be the same for data of either kind. We therefore determine both D_p and D_v (from photographic and photovisual data, respectively), and with the aid of (12) obtain an equation involving A_p and A_v, their derivatives, and the distance ρ. Since any selective absorption will almost certainly be the result of scattering, A_v and A_v' can be expressed in terms[2] of A_p and A_p', thus leaving a first-order differential equation in A with coefficients involving ρ, which can be solved numerically by some method of approximation, if not analytically. This procedure, however, is a matter for the future when observational data have become more complete, and for the present A must be found independently by any method practicable.

As for D, several functional forms have been proposed, each of which, within certain limits, may be used to represent the apparent

[1] *Astronomische Nachrichten*, **185**, 81, 1910.

[2] For the international color system the ratio of A_p to A_v for scattering by Rayleigh's law is approximately 2. For particles of appropriate size other values may, however, be involved.

distribution of density for the stars as a whole. The most useful, however, except when dealing with stars close to the sun, is Schwarzschild's analytical solution of (11) for the case in which N and ϕ are both quadratic exponentials. The form is then

$$D(\rho_0) = e^{h+k\,(\log \rho_0)+l\,(\log \rho_0)^2}, \tag{13}$$

where h, k, and l depend on similar coefficients in the exponentials for N and ϕ.[1] Since k is positive and l negative, (13) fails for small values of the distance, but for most purposes this defect is not serious. The difficulty originates in the expressions for N and ϕ. A quadratic exponential in m_0 represents satisfactorily the smoothed star counts from the eleventh or twelfth magnitude downward, but the brighter and hence relatively near stars are discordant.[2]

The form of ϕ, which for the present must be regarded as the same at all distances, does not seem to be a quadratic exponential,[3] but for all stars having $M < 11.5$ (international zero-point) it may be regarded as such. The exceptions are practically negligible, for they do not appear in the counts of stars brighter than photographic magnitude 18.5 unless their distances are less than 250 parsecs. The failure of the exponentials for both N and ϕ therefore affects the nearby stars, and in consequence (13) is also defective for these stars. The representation beyond the limit set by $\log \rho = h/2l$ should, however, be good. Within this limit, and indeed considerably beyond it, the simple expression

$$D(\rho_0) = \alpha e^{-\beta \rho_0} \tag{14}$$

can often be used.

These conclusions apply to the case in which the influence of star clouds and other irregularities has been eliminated from the star counts by smoothing. Consequently the resulting values of D for any direction in the galactic plane must also be very much smoothed, as the form of (13) itself shows. Such a method of procedure is

[1] Kapteyn and van Rhijn, *Mt. Wilson Contr.*, No. 229; *Astrophysical Journal*, **55**, 242, 1922.

[2] *Mt. Wilson Contr.*, No. 301, Table XXIII; *Astrophysical Journal*, **62**, 320, 1925.

[3] *Mt. Wilson Contr.*, No. 273; *Astrophysical Journal*, **59**, 310, 1924; van Rhijn, *Groningen Publication*, No. 38, 1925.

very unsatisfactory because it leads to no detailed knowledge of the structure of the stellar system. It has, however, given us useful general indications as to stellar distribution, and we may use it once more to obtain general information as to the consequences of absorption.

Finally, before using (13) for calculation of the effect of absorption, it is important to know whether the adopted expression for ϕ has itself been influenced by the presence of absorbing material. This seems unlikely since most of the stars on which ϕ is based are well within 1000 parsecs of the sun. Moreover, these stars are scattered over the whole sky, while absorption seems to be confined to a thin stratum close to the galactic plane. Any influence must therefore be small.

2. APPLICATIONS

The substitution of equation (13) into (12) gives for the density, freed from the effect of absorption, the alternative expressions

$$\log \Delta(\rho) = h' + k' (\log \rho + A) + l' (\log \rho + A)^2 + 3A + B , \qquad (15a)$$

$$\log \Delta(\rho) = \log D(\rho) + KA + l'A^2 + B , \qquad (15b)$$

where

$$K = 3 + k' + 2l' \log \rho , \qquad B = \log \left(1 + \rho \frac{A'}{\mu}\right) , \qquad \mu = 0.4343 , \qquad (16)$$

and where h', k', and l' are the coefficients of (13) multiplied by the modulus. These formulae apply to distances beyond the limit set by $\log \rho = k/2l$, which is usually less than 100 parsecs.

For the region near the sun, (14) and (12) give

$$\log \Delta(\rho) = q \log D(\rho) + 3A + B , \qquad \log q = A . \qquad (17)$$

In both (15) and (17), $D(\rho)$ represents numerical values calculated with (13) and (14), respectively, for specified values of the true distance ρ. Although (13) and (14) are functions of the fictitious distance, the distinction between ρ and ρ_0 has hitherto been ignored. Consequently values of D already available may be used directly in (15) and (17). All logarithms are to the base 10.

Our knowledge of the absorption function A is so limited that we

are obliged to assume different forms and study the resulting effect on Δ. The various assumptions tested are indicated by the second column of Table I. The first of these corresponds to the results of Trumpler and of van de Kamp,[1] who accept an absorption coefficient which (in the galactic plane) is independent of the distance; A is therefore proportional to ρ. The second assumption, superficially at least, seems to have a certain plausibility in that it supposes the absorption coefficient at each point to be proportional to the true density Δ at that point. This coefficient increases toward the center of the system. The third assumes proportionality with D instead of

TABLE I

ASSUMPTIONS FOR A

DESIGNATION IN TABLE II	A	A'		Δm, INTERVAL 0–1000 PARSECS	
				$\lambda = L$	$\lambda = 180° + L$
Δ_1	$\epsilon_1 \rho$	ϵ_1	0.0001340	0.67	0.67
Δ_2	$\epsilon_2 \int^{\rho} \Delta d\rho$	$\epsilon_2 \Delta$	.0000848	.57	.26
Δ_3	$\epsilon_3 \int^{\rho} D d\rho$	$\epsilon_3 D$	.0001696	.71	.46
Δ_4	$\epsilon_4 \int^{\rho} D d\rho$	$\epsilon_4 D$	0.0000848	0.36	0.23

Δ and thus uses an absorption coefficient which decreases as ρ increases. The last retains the form of No. 3, but uses a numerical coefficient one-half as great.

The values of Δ have been calculated by (15) and (17) for two directions in the galactic plane, one toward the center of the system in Sagittarius ($\lambda = L$), the other in the opposite direction. For the first three assumptions ϵ is such that the value of $\Delta m = 5A$ for the first thousand parsecs in the direction $\lambda = L$ is equal, approximately at least, to Trumpler's coefficient of 0.67 mag. (photographic light). The exact values of Δm are in the last two columns of Table I. The data for $D(\rho)$ are from *Mt. Wilson Contr.* No. 347, equations (63) and Table VII,[2] and are based on Mount Wilson counts of photographic magnitudes. The calculation is straightforward except for the second case. Since the unknown Δ here appears under the inte-

[1] *Loc. cit.* [2] *Astrophysical Journal*, **67**, 123, 1928.

gral in A, approximations must be used. A step-by-step calculation from the origin outward gives, however, a very rapid convergence.

The results are in Table II and are illustrated[1] in Figure 1. The left half of the diagram shows that almost any value of the absorption means at least a temporary increase in Δ toward the center of the system. The very marked increase in this direction corresponding to Trumpler's result of 0.67 mag. per thousand parsecs (Δ_1) is

TABLE II

EFFECT OF ABSORPTION ON SPACE DENSITY IN THE GALACTIC PLANE

ρ (PARSECS)	$\lambda = L$					$\lambda = 180° + L$				
	D	Δ_1	Δ_2	Δ_3	Δ_4	D	Δ_1	Δ_2	Δ_3	Δ_4
0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
100	0.99	1.10	1.05	.....	.....	0.93	1.05	1.00	1.07	1.00
200	0.96	1.20	1.10	.....	.....	0.76	0.96	0.89	0.97	0.86
300	0.92	1.30	1.15	.....	.....	0.65	0.89	0.79	0.89	0.76
400	0.87	1.40	1.22	.....	.....	0.55	0.83	0.69	0.80	0.67
500	0.82	1.51	1.29	1.62	1.16	0.47	0.79	0.58	0.71	0.58
1000	0.70	2.09	1.93	2.06	1.22	0.28	0.68	0.39	0.48	0.37
1500	.....	.....	6.30	2.30	1.19	.....	.....	0.28	0.34	0.25
2000	0.49	3.09	.....	2.27	1.09	0.14	0.65	0.20	0.26	0.19
3000	0.35	3.98	.....	1.99	0.88	0.08	0.67	0.13	0.16	0.12
4000	0.27	4.79	.....	1.68	0.71	0.06	0.72	0.094	0.12	0.085
5000	0.21	5.37	.....	1.33	0.56	0.045	0.74	0.083	0.083	0.062
6000	0.17	5.62	.....	1.09	0.47	0.03	0.78	.....	.....	.....
7000	0.14	5.50	.....	.....	.....	0.025	0.78	.....	.....	.....
8000	0.11	5.13	.....	.....	.....	0.02	0.74	.....	.....	.....
9000	0.10	4.47	.....	.....	.....	0.02	0.70	.....	.....	.....
10000	0.09	3.63	.....	.....	.....	0.01	0.51	.....	.....	.....
20000	0.03	2.40	.....	.....	.....	.....	.....	.....	.....	.....

not necessarily impossible, although the rate of increase may be too large. In fact, one of the puzzling things about star counts is that superficially they give so little evidence of an increase in space density in the direction of the center. The probable influence of absorbing material in depressing the calculated density has been recognized, but quantitative relations have not hitherto been available.[2] The curves suggest that a moderate amount of absorption will account completely for the anomaly shown by the star counts. The exact form of the function A naturally remains uncertain. The decreasing functions (A_3, A_4) are scarcely applicable, however, since

[1] The values of Δ_4 for $\lambda = 180° + L$ are so nearly equal to those for Δ_4 in the same direction that they are not plotted in Fig. 1.

[2] *Mt. Wilson Contr.*, No. 347, sec. 13; *Astrophysical Journal*, **67**, 170, 1928.

the corresponding maxima in Δ occur at points which seem to be too close to the sun. On the other hand, a coefficient proportional to Δ, at least one with a numerical coefficient as large as that used for Δ_2, is impossible; the resulting rapid increase in A is wholly inadmissible. Perhaps the most probable coefficient is one which initially is somewhat smaller than Trumpler's and which slowly increases toward the center of the system. An increase in this direction is rather to be expected; and such a coefficient would shift the maximum in Δ_1

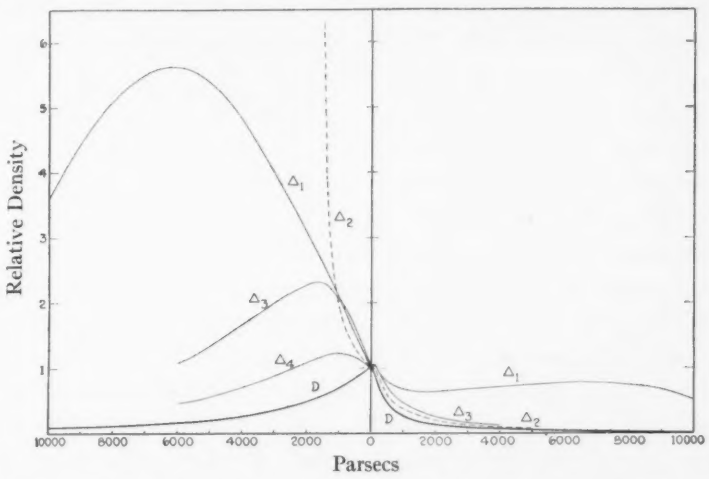


FIG. 1.—Effect of absorption on space density of stars in the galactic plane: left, toward the center of the system in $\lambda = 325°$; right, toward $\lambda = 145°$. D, mean relative densities found from Mount Wilson star counts by ignoring absorption. Δ_1 corresponds to a constant absorption of 0.67 mag. per 1000 parsecs. The remaining curves refer to other coefficients (Table I).

farther outward and thus harmonize better with generally accepted values of our distance from the center. Moreover, a smaller initial value would help avoid a difficulty which appears when other directions in the galactic plane are considered.

For $\lambda = 180° + L$ the densities which correspond to Trumpler's constant coefficient seem improbable; for $\lambda = 90° + L$ and $\lambda = 270° + L$ they appear to be quite impossible. Calculations have not been made for these intermediate directions, but it is clear that Δ would show a conspicuous increase with increasing distance, since its values in these directions must be roughly the mean of those toward the center and anticenter. An increase in Δ toward the center of the system is

admissible, but certainly not in other directions, at least not in the smoothed densities. A slightly smaller coefficient which decreases slowly as ρ increases seems therefore to be demanded for all longitudes except those in the general direction of the center.

On the whole, therefore, it does not seem that the absorption coefficient can be constant over any great distance for any direction in the galactic plane. Toward the center an increase is perhaps to be expected, and, if actually present, would explain why counts of stars in this direction give so little indication of increasing space density. A coefficient corresponding to an absorption of 0.67 mag. per thousand parsecs seems rather high; the 0.8 mag. found by Trumpler from his discussion of the diameters of open clusters is even more in question[1] because of the difficulty of avoiding an increase in density in directions for which an increase seems inadmissible.

These conclusions of course refer to smoothed values of the absorption, just as the densities themselves are smoothed and take no account of such striking features as the galactic star clouds, and are admittedly provisional in so far as they may be affected by irregularities in the distribution of absorbing material. These irregularities undoubtedly are large. The material producing the absorption must be finely divided dust particles rather than the molecules of a continuous stratum of gas, for otherwise difficulties with the total mass involved would be insuperable. It is natural, therefore, to associate absorption with the obscuring clouds, whose irregularity of distribution is obvious. Most of the regions in which selective absorption has been detected are within or close to the boundaries of such clouds, and it is possible that the mean observed absorption is larger than that which would be found if the fields investigated were impartially distributed over both obscured and unobscured regions. This remark has a bearing on the coefficient found from stars showing a color excess, but does not explain the large value found by Trumpler in his discussion of the diameters of open clusters, unless absorbing material tends to concentrate about the clusters to a greater extent than elsewhere.

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[1] *Lick Observatory Bulletins*, 14, 164, 167 (No. 420), 1930.

THE SPECTROSCOPIC ORBIT OF RT LACERTAE[1]

By ALFRED H. JOY

ABSTRACT

Spectroscopic observations and orbit.—The brighter component of the eclipsing variable RT Lacertae is G9 and the fainter K1, but the latter has the greater mass. The orbital elements based on 22 spectrograms (Table I) are: $e=0.0$; $\gamma_1=-47.0$, $\gamma_2=-47.0$, $K_1=62.5$, $K_2=116.0$ km/sec.; $a_1 \sin i=4{,}360{,}000$, $a_2 \sin i=8{,}090{,}000$ km; $m_1 \sin^3 i=1.9$, $m_2 \sin^3 i=1.0$ ⊙.

Absolute dimensions.—Photometric results from Miss Fowler's solution combined with the spectroscopic elements give the absolute dimensions: $a_1=4{,}360{,}000$, $a_2=8{,}090{,}000$, $a_1+a_2=12{,}450{,}000$ km; $r_1=r_2=5.0$, $b_1=b_2=4.8$ ⊙; $m_1=1.9$, $m_2=1.0$ ⊙; $\rho_1=0.02$, $\rho_2=0.01$ ⊙; $M_1=3.5$; $M_2=3.0$; $\pi=0''.005$.

RT Lacertae (α $21^h57^m.5$, δ $+43°24'$, 1900; G5, 8.8–9.8 mag.) is an eclipsing star of late spectral type, discovered by Mme L. Ceraski in 1908. A limited number of visual photometric observations have been reported by S. Enebo,[2] M. Luizet,[3] and A. A. Nijland.[4] H. Shapley[5] and Miss M. Fowler[6] have determined approximate photometric orbits from the measures of Enebo and Luizet.

Twenty-three spectrograms of RT Lacertae were obtained with the 100-inch reflector between 1919 and 1928. The data concerning the spectrograms, measured radial velocities, weights, and residuals are listed in Table I. The phases were computed from Nijland's elements

$$\text{Min.} = \text{J.D. } 2418204.444 \text{ G.M.T.} + 5^d.073921E ,$$

which are based on the most recent published observations, some of which were made during the period covered by the spectrographic observations. Although most of the spectrograms were made with the low dispersion of 7- and 10-inch cameras, the lines of both com-

[1] *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 429.

[2] *Astronomische Nachrichten*, **180**, 360, 1909; **184**, 396, 1910; *Beobachtungen veränderlicher Sterne*, **4**, 46; *Archiv för Mathematik og Naturvidenskab*, B, **30**, No. 12, 1910.

[3] *Bulletin astronomique*, **27**, 302, 1910; **32**, 68, 1915.

[4] *Astronomische Nachrichten*, **204**, 65, 1917; **211**, 361, 1920; *Bulletin of the Astronomical Institutes of the Netherlands*, **2**, 130, 1924.

[5] *Contributions from the Princeton University Observatory*, **3**, 86, 171, 1915.

[6] *Astrophysical Journal*, **52**, 257, 1920.

ponents are measurable on fourteen plates. The type of the brighter star is G9, while that of the fainter is somewhat later, K1. The lines of both spectra are fairly good, but are considerably blurred by the overlapping continuous background and the blending of adjacent lines. In the case of double-lined spectra of late type taken with low dispersion it is not possible to eliminate errors of measurement

TABLE I

OBSERVATIONS OF RT LACERTAE

PLATE	G.M.T.	PHASE	CAMERA	PRIMARY			SECONDARY		
				Vel.	Wt.	O−C	Vel.	Wt.	O−C
	J.D. 242+	days	in.	km/sec.		km/sec.	km/sec.		km/sec.
C 759	2649.710	0.512	18	− 3.1	0.2	+ 6.9	−109.1	0.7	+ 6.6
1362	2969.735	0.879	18	− 1.2	0.7	− 9.6	−156.5	1.0	− 6.7
4409	5131.678	1.332	18	+ 20.0	0.2	+ 4.7	−164.8	0.5	− 2.2
1123	2883.967	1.368	7	+ 11.7	0.5	− 3.3	−153.8	0.7	+ 8.3
3067	4096.635	1.369	10	+ 23.9	0.2	+ 8.9	−164.5	0.5	− 2.4
3887	4715.937	1.653	10	+ 28.8	0.2	+20.2	−149.6	0.7	+ 0.5
1805	3269.932	1.715	7	+ 15.3	0.1	+ 9.1	−143.5	0.5	+ 2.2
1267	2940.784	2.372	7	.....	.....	.....	− 63.4	0.5	+ 7.2
4389	5127.802	2.530	10	.....	.....	.....	− 61.3	0.5	−12.7
769	2651.733	2.535	18	.....	.....	.....	− 51.1	0.2	− 3.8
2932	4011.694	2.685	10	.....	.....	.....	− 24.0	0.5	+ 1.9
2481	3712.646	2.998	10	.....	.....	.....	(− 19.2)	.....	.....
167	2266.713	3.132	18	− 94.3	1.0	− 5.3	+ 28.0	1.0	− 3.0
1970	3357.667	3.193	7	.....	.....	.....	(− 18.6)	.....	.....
2992	4037.860	3.480	10	−101.8	0.5	+ 2.7	+ 46.3	0.5	−13.4
4632	5255.602	3.482	10	−104.8	0.2	0.0	+ 52.3	0.5	− 7.2
3517	4418.696	3.773	18	−101.2	1.0	+ 8.6	+ 69.4	1.0	+ 0.1
2484	3713.637	3.989	10	− 97.0	0.1	+10.9	+ 66.7	0.2	+ 0.7
2570	3769.668	4.207	10	−134.2	0.1	−32.3	+ 55.8	0.2	+ 0.8
1843	3297.833	4.246	18	−106.1	0.5	− 5.7	+ 53.6	1.0	+ 1.5
3973	4749.774	5.046	10	− 39.0	0.5	+10.2	.....	.....	.....
2298	3592.922	5.048	7	− 49.1	0.5	− 0.1	.....	.....	.....

due to blending of partially superposed lines; but when a considerable number of lines is measured, such errors should not, in the mean, prove unduly dangerous. The average number of lines used is 8.5 per plate for the primary spectrum and 12.0 for the secondary. The lines of the brighter secondary component are about three times as strong as those of the fainter primary. It is very unusual for the fainter component of a spectroscopic binary to be more massive than the brighter.

The individual measures (plotted in Fig. 1) of the velocities of

both components were used with the weights given in Table I for a least-squares solution to determine the elements of the orbits of the components. The solution was carried out by Mr. N. U. Mayall. Spectrograms C 1970 and C 2481 were omitted because the lines of the two components are not sufficiently separated for measurement. C 4632, taken after the solution was made, was therefore not in-

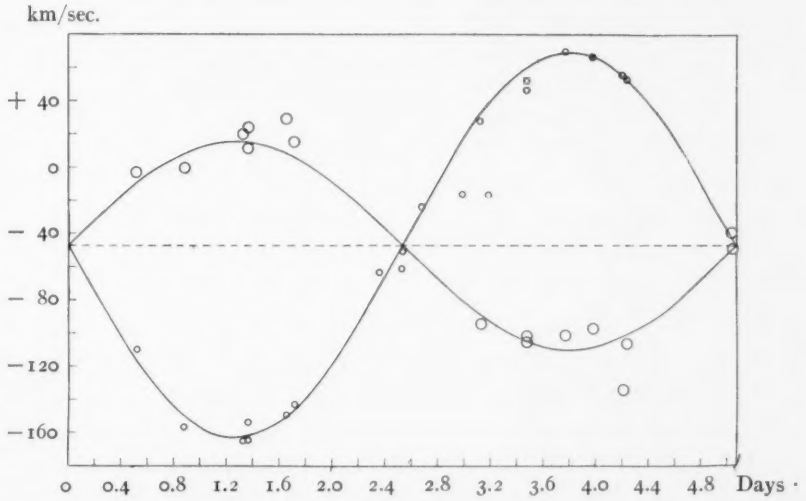

FIG. 1.—Velocity-curve of RT Lacertae. The larger circles represent the observed velocities of the fainter primary star and the smaller circles those of the brighter secondary.

cluded. The elements derived and the corresponding probable errors are:

$e=$ 0.0 (assumed)	$a_1 \sin i = 4{,}360{,}000$ km
$\gamma_1 = -47.0 \pm 1.49$ km/sec.	$a_2 \sin i = 8{,}090{,}000$
$\gamma_2 = -47.0 \pm 1.09$	$m_1 \sin^3 i = 1.9\ \odot$
$K_1 = 62.5 \pm 1.86$	$m_2 \sin^3 i = 1.0$
$K_2 = 116.0 \pm 1.38$	

The probable errors for an observation of velocity of weight unity are 3.81 and 3.25 km/sec. for the primary and secondary, respectively.

Using $i = 89°10'$, $a_b = a_f = 0.277$, and $b_b = b_f = 0.267$ from Miss

Fowler's solution[1] for stars darkened at the limb, we find the following absolute dimensions:

a_1, semi-major axis of primary orbit	4,360,000 km
a_2, semi-major axis of secondary orbit	8,090,000
a_1+a_2, semi-major axis of relative orbit	12,450,000
r, longer radii of both stars	3,450,000 (5.0 ⊙)
b, shorter radii of both stars	3,320,000 (4.8)
m_1, mass of fainter star	1.9 ⊙
m_2, mass of brighter star	1.0
ρ_1, density of fainter star	0.02 ⊙
ρ_2, density of brighter star	0.01
M_1, absolute magnitude of fainter star	3.5
M_2, absolute magnitude of brighter star	3.0
π, parallax	0".005

The absolute magnitudes were computed from the formula

$$M=\frac{29,500}{T}-5\log r-0.08\,,$$

with $T=4500°$ and $4200°$ for the brighter and fainter stars, respectively. The selected temperatures are intermediate between those of giants and dwarfs. The spectroscopic absolute magnitudes determined with the co-operation of W. S. Adams are 3.0 and 2.5. The parallax is from the computed absolute magnitudes and the observed apparent magnitudes 9.4 and 9.8. A. van Maanen[2] has found an absolute trigonometric parallax of 0".027.

The stars are too faint for typical giants and the densities are intermediate between those of giants and dwarfs. Since the fainter star has the greater mass, it is impossible to reconcile the results with Eddington's mass-luminosity correlation although the individual discordances are not excessive.

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[1] *Op. cit.*, p. 261.

[2] *Mt. Wilson Contr.*, **16**, 248 (No. 356), 1928.

PHOTOMETRIC CONSEQUENCES OF THE GROWTH OF THE LATENT IMAGE

By PHILIP C. KEENAN

ABSTRACT

The opinion of investigators of the subject favors the view that the latent image continues to increase in intensity for a few hours after exposure and then tends to fall off again. Tests to determine the practical photometric consequences of this effect on ordinary plates show that increases of density of the order of 10 per cent occur after two hours; hence, serious errors in precise photometric comparisons may result if this is not taken into account.

A recent study by N. Barabascheff and B. Semejkin[1] of changes in the photographic latent image between exposure and development gave evidence of a consistent strengthening of the image, greatest during the first two days after exposure. The magnitude of this growth, termed by the authors *Akkumulationseffekt*, amounted under extreme conditions to a difference of density of 0.2 and in general represented a considerable proportion of the total density.

Other investigations of the subject have given inconsistent results, as was to be expected from the variety of the photographic materials employed. Whereas Barabascheff and Semejkin speak almost entirely of a positive effect, such earlier workers as H. J. Channon[2] and L. Baekeland[3] found a diminution of the strength of the image, or photo-regression, as the most general result of delayed development. However, the time intervals involved were of the order of days, and C. F. Brush pointed out in 1910 that "photographic action, when fully excited, continues many minutes after exposure, gradually dying out. With the adopted period of exposure, this after-action amounts to something like 8 or 10 per cent of the whole. Within an hour or two after action has ceased, relapse sets in and amounts to about 4 per cent in the first thirty hours, fully half of which occurs within the first four hours."[4]

[1] *Zeitschrift für wissenschaftliche Photographie, Photophysik, und Photochemie*, **28**, 333, 1931.

[2] *Photographic Journal*, **57**, 72, 1917.

[3] *Zeitschrift für wissenschaftliche Photographie, Photophysik, und Photochemie*, **3**, 58, 1905.

[4] *Physical Review*, **31**, 241, 1910.

This phenomenon of a continued growth immediately after exposure, diminishing with time, has been confirmed for normal densities by the more exact quantitative experiments of G. Jausseran[1] and of E. R. Bullock[2] on a number of European and American plates, although the former concluded that the medium densities approach a constant value after several hours instead of actually diminishing again. The maximum increase of density obtained by him was 0.50. Bullock's extensive researches emphasized the complicated nature of the effect and demonstrated that it is distinct from any influence of accidental humidification of the plates between exposures.

Contradictory as are these several sets of data, it is nevertheless evident from them that the time allowed to elapse between exposure and development can have an appreciable effect upon photometric measurements of a plate. Although rather elaborate investigations will be necessary to furnish a complete description of the phenomenon, and any experimenter employing precise photometry will probably have to test for its presence under his particular working conditions, it is useful to have some information as to its practical influence on astronomical plates taken with some such ordinary procedure as the impressing of sensitometer exposures several hours, or even days, before the plates are to be used. To this end a set of tests on commercial plates used extensively at the Yerkes Observatory was suggested by Professor Ross.

The test exposures were made with a sensitometer, which is fed by a storage battery which was found to keep the illumination satisfactorily constant over the duration of each series. Two exposures separated by the desired time interval were made on each plate, which was developed immediately after the second. Pairs of plates were usually taken in duplicate and developed simultaneously in order to detect any large accidental errors.

The first interval of time tested was two days, Eastman 40 and

[1] *Comptes rendus*, **188**, 783, 1929.

[2] *Science et industries photographiques*, **1**, 124, 169, 321, 366, 1930. Eastman Kodak Company, Abridged Scientific Publications, Communication No. 425. The abridgment was kindly sent in advance of publication by the author to Professor Ross, who placed it at my disposal.

Imperial Eclipse Soft emulsions being used. Eight plates of each type were taken, with an additional one bearing exposures made at the beginning and the end of the series to serve as a check. Neither of the two series showed consistent differences between the first and second sets of standard squares, so, while a slight strengthening of the earlier images was suspected, it is not established and can scarcely amount to a difference of density of as much as 0.03 for moderate blackenings. In view of these negative results detailed measurements were not attempted.

Eight Eastman 40 plates were then tested for an interval of two hours, approximately the time required for the density to increase to its maximum value according to the curves of Jausseran, and here

TABLE I

INCREASE OF DENSITY IN AN INTERVAL OF TWO HOURS

Hole	log L	D_1	D_2	D_1-D_2	P.E.	Per Cent Diff.
1	1.9	0.88	0.86	0.02	0.003	2
2	1.6	.70	.65	.05	.002	7
3	1.3	.40	.36	.04	.002	12
4	1.0	.22	.20	.02	.002	9
5	0.7	.078	.070	.008	.0008	11
6	0.4	0.015	0.013	0.002	0.0016	15

the first set of exposures showed greater blackening over most of the range of density on every plate. The images were measured on a Hartmann visual photometer, the results being averaged in Table I. The columns represent, in order, the designation number of the sensitometer square, the logarithm of the relative intensity, the density of the first exposure, the density of the second, the difference between the two, the probable error of the difference, and the difference expressed as a percentage of the density of the second exposure.

The effect can be seen more clearly by considering the gain in effective intensity of the light due to a delay of two hours before development. The percentage gain for different densities is shown in Table II.

It may be observed that the increase in density is greatest for moderately strong blackenings, a result in agreement with the find-

ings of Jausseran, whose conclusion that the characteristic curve is made steeper by delaying development is also confirmed here, although the difference in the average values of γ_{max} amounts to only about 0.02. To judge of the practical effect of the change in the curve, consider a continuous spectrum with an absorption line having a central intensity of 20 per cent. Let the density produced by the continuous spectrum be 0.6. If the plate is developed immediately after exposure to the spectrum, the sensitometer squares having been impressed about two hours previously, the residual intensity in the line will be measured as 21 per cent.

TABLE II

GAIN IN EFFECTIVE INTENSITY

DENSITY	INCREASE IN INTENSITY	DENSITY	INCREASE IN INTENSITY
	Per Cent		Per Cent
0.1	4	0.5	12
.2	10	.6	12
.3	12	0.7	12
0.4	12		

Although the growth observed here is quite definite, the differences of density are much smaller than those obtained by some of the authors cited above, particularly Barabascheff and Semejkin. It is difficult to say whether this is due to the choice of time intervals, the type of plate tested (since emulsions of higher contrast would be expected to show the effect more clearly), or to other peculiarities in the individual working conditions. Additional factors, such as the humidity effect discussed by Bullock, may easily come into play and give rise to spurious results.

In general it appears safe to conclude that changes in the latent image before development will normally produce differences of density of only a few hundredths of a unit, but under exceptional conditions the amount may rise to much greater values. For photometry of any precision, images of which the densities are to be compared directly should be developed at as nearly the same time after exposure as possible. In the case of sensitometer squares used only

to provide a calibration scale the errors involved will usually be small enough to be neglected in view of the greater convenience of making the calibration exposures in advance, though it would be best to do so at least a day ahead rather than only a few hours before the final exposure, provided development is to follow the latter immediately.

YERKES OBSERVATORY
May 1931

THE DISTRIBUTION OF ABSOLUTE MAGNITUDES AMONG F AND G STARS BRIGHTER THAN THE SIXTH APPARENT MAGNITUDE AS DETERMINED FROM PARALLACTIC AND PECULIAR VELOCITIES[1]

By GUSTAF STRÖMBERG

ABSTRACT

1. The distribution of *absolute magnitudes* among F and G stars brighter than apparent magnitude 6.0 has been determined from the distribution of parallactic and peculiar reduced proper motions and of radial velocities by methods described in *Mount Wilson Contributions* Nos. 395 and 410. The number of proper-motion stars in the F and G groups is 622 and 601, respectively; of radial-velocity stars, 573 and 707, respectively.

2. The distributions of absolute magnitudes are shown in Table V and Figs. 5 and 6. Among stars of types Fo–F9 the distribution shows three distinct maxima: *supergiants* between $M=-5$ and -2.0; *normal giants* between $M=-0.2$ and $+2.2$; and *dwarfs* or *faint giants* between $M=+2.2$ and $+5.0$. The relative proportion of stars in the three groups is 9, 45, and 46 per cent. The distribution for the spectral interval Go–G9 reveals three distinct maxima: *supergiants* between $M=-5$ to -1.5; *normal giants* between $M=-0.5$ and $+1.5$; and a third group between $M=+1.5$ and $+5.5$. The relative proportion of stars in the three groups is 19, 49, and 32 per cent. The third group consists possibly of *faint giants* with a maximum frequency at $M=+3.0$ and of *dwarfs* at $M=+4.0$, in a relative proportion of 23 and 9 per cent. Among both the F and the G stars there is a *complete absence of stars* having an absolute magnitude near $M=-1.0$.

3. The distributions of reduced angular parallactic and peculiar motions and of the corresponding linear motions are given in Tables II–IV.

The methods for determining the distribution of absolute magnitudes from peculiar and parallactic motions outlined in *Mount Wilson Contributions* Nos. 395[2] and 410[3] have been applied to the F and G stars brighter than the sixth apparent magnitude, with the results given in this paper. The final results for the M stars are given in *Contribution* No. 411,[4] and for the K stars in *Contribution* No. 418.[5]

All stars assigned in the *H.D. Catalogue* to types F and G, respectively, and brighter than the sixth apparent magnitude were included. To these F and G stars were added those similarly classified by the Mount Wilson observers. Several stars classified differently in the

[1] *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 430.

[2] *Astrophysical Journal*, **71**, 163, 1930.

[3] *Ibid.*, **72**, 111, 1930. [4] *Ibid.*, p. 117, 1930. [5] *Ibid.*, **73**, 40, 1931.

H.D. Catalogue and at Mount Wilson thus appear in both groups. Further, some stars designated in the *H.D. Catalogue* as A and K but at Mount Wilson as F and G, respectively, are also included. The rules for excluding variables and double stars applied in the case of the K and M stars were also used in the present study.

The total number in the group of F stars is 622 and in that of G stars, 601. Seventeen G stars of large proper motion and of well-determined parallax were excluded from the analysis, although they appear in the tables showing the distribution of peculiar and parallactic motions, and their absolute magnitudes were afterward added to the calculated distribution.

TABLE I

F0–F9				G0–G9			
M_S	No.	V_0	θ	M_S	No.	V_0	θ
		km/sec.	km/sec.			km/sec.	km/sec.
≤0.0.....	32	21.3	15.1	≤−0.6....	25	26.3	11.6
+0.1 to +2.2..	193	22.1	14.1	−0.5 to +0.5.	266	17.9	14.9
+2.3 to +3.2..	162	19.1	15.1	+0.6 to +2.4.	213	16.3	20.2
+3.3 to +6.0..	186	17.1	18.3	+2.5 to +4.4.	118	23.1	25.8
				+4.5 to +6.5.	85	39.1	21.7

In the analysis of parallactic motions, 90 F-type and 85 G-type stars, whose angular distances from the apex or antapex were less than 30°, were also excluded. The two spectral groups include, respectively, 573 and 707 stars brighter than the seventh apparent magnitude for which both spectroscopic absolute magnitudes and radial velocities are available. These were used for determining the distributions of linear motion corresponding to peculiar and parallactic angular motions. As a preliminary, the group motion V_0 and the average radial velocity θ were determined for several groups divided on the basis of spectroscopic absolute magnitudes. The results appear in Table I. These values for V_0 were used to determine the linear velocities corresponding to the parallactic motions. The values for θ were not used, the actual distribution of peculiar velocities being utilized.

The distributions of

$$y_1 = \log|\epsilon| - 1.6756\,,$$
$$y_2 = \log|V_0 + \epsilon/\sin\lambda| - 1.6756\,,$$

where ϵ is the peculiar radial velocity and λ the angular distance from the sun's apex, are given in Tables II and III. The values of

TABLE II

Fo–F9

y	$F(y_1)dy$				$F(y_2)dy$			
	+0.2	+2.2	+3.7		+0.2	+2.2	+3.7	
−3.5	.....	.....	.....	.....	.....	.....	1	.....
−3.4	1	.....	.....	.....	1	1	0	.....
−3.3	0	1	1	.....	0	0	0	.....
−3.2	0	0	0	1	0	0	0	.....
−3.1	1	0	0	0	0	0	0	1
−3.0	1	1	1	0	0	0	1	0
−2.9	1	1	1	1	0	1	0	0
−2.8	1	1	1	1	1	0	1	0
−2.7	2	1	1	1	0	1	1	1
−2.6	2	1	1	1	0	1	1	0
−2.5	2	2	2	1	1	1	1	1
−2.4	3	2	2	2	1	1	2	1
−2.3	4	3	3	2	1	1	2	1
−2.2	5	3	3	3	1	2	2	1
−2.1	6	4	4	4	2	2	3	1
−2.0	8	6	5	5	2	3	4	2
−1.9	10	7	6	6	3	3	5	2
−1.8	13	9	8	8	3	4	6	3
−1.7	15	11	10	9	4	5	8	4
−1.6	19	14	13	12	5	7	10	5
−1.5	24	18	16	15	7	8	12	6
−1.4	29	21	20	18	8	10	14	8
−1.3	35	25	25	23	10	13	17	10
−1.2	42	30	29	30	12	16	20	14
−1.1	56	36	35	39	14	19	23	19
−1.0	71	44	43	47	17	24	29	26
−0.9	77	71	51	56	20	30	35	35
−0.8	81	90	75	65	29	40	44	46
−0.7	83	100	107	80	62	55	56	65
−0.6	84	105	118	93	93	68	73	89
−0.5	83	106	119	96	112	82	88	105
−0.4	79	105	117	97	142	106	100	122
−0.3	71	102	85	95	158	140	111	117
−0.2	60	58	52	82	112	132	118	102
−0.1	30	17	30	55	80	106	117	80
0.0	1	5	12	23	58	76	58	62
+0.1	.....	.....	4	14	30	35	23	38
+0.2	.....	.....	.....	9	10	6	10	17
+0.3	.....	.....	.....	5	1	1	3	9
+0.4	.....	.....	.....	1	.....	.....	1	5
+0.5	.....	.....	.....	.....	.....	.....	.....	2
+0.6	.....	.....	.....	.....	.....	.....	.....	.....
$\bar{y}$	−0.792	−0.698	−0.670	−0.612	−0.407	−0.419	−0.480	−0.412

TABLE III

G0–G9

y	$F(y_1)dy$					$F(y_2)dy$				
	−1.0	+0.8	+2.2	+4.2		−1.0	+0.8	+2.2	+4.2	
−3.7	1	.....	.....	.....	.....	.....	.....	.....	.....	.....
−3.6	0	1	.....	.....	.....	.....	.....	.....	.....	.....
−3.5	0	0	.....	.....	1	.....	.....	.....	.....	.....
−3.4	0	0	1	.....	0	.....	.....	.....	.....	.....
−3.3	1	0	0	.....	0	.....	1	.....	.....	.....
−3.2	0	1	0	1	0	.....	0	1	1	.....
−3.1	1	0	0	0	1	.....	0	0	0	.....
−3.0	0	0	1	0	0	1	0	0	0	1
−2.9	1	1	0	0	1	0	1	1	0	0
−2.8	1	1	1	1	1	0	0	0	1	0
−2.7	1	1	1	0	1	0	1	0	0	0
−2.6	1	1	1	1	1	1	0	1	1	1
−2.5	2	2	1	1	1	0	1	1	1	0
−2.4	2	2	1	1	2	0	1	1	1	0
−2.3	2	3	2	1	2	1	1	1	1	1
−2.2	3	3	2	1	3	1	2	2	2	1
−2.1	4	4	3	2	4	1	2	2	2	1
−2.0	5	5	4	2	5	1	2	3	3	1
−1.9	7	6	5	3	6	2	3	3	3	1
−1.8	9	8	6	4	8	2	4	4	4	2
−1.7	11	10	7	5	9	3	5	5	5	2
−1.6	13	13	9	6	12	3	7	6	6	3
−1.5	17	17	11	8	15	4	8	8	8	4
−1.4	21	21	13	10	19	5	10	10	9	5
−1.3	45	27	16	13	23	6	13	13	11	6
−1.2	80	37	23	18	31	7	16	16	13	7
−1.1	92	56	31	27	37	11	20	21	15	9
−1.0	93	66	40	36	43	14	28	28	20	11
−0.9	92	78	52	43	50	18	39	35	24	13
−0.8	89	88	66	52	58	23	53	45	33	16
−0.7	83	92	89	64	65	29	70	56	42	21
−0.6	77	93	100	78	73	44	89	67	58	28
−0.5	71	90	101	96	79	102	108	84	68	42
−0.4	62	84	100	117	82	140	130	104	89	64
−0.3	48	74	97	122	83	180	123	109	97	109
−0.2	39	58	83	94	82	164	103	108	99	138
−0.1	24	30	65	70	75	116	68	98	99	141
0.0	2	14	40	50	66	76	43	78	96	141
+0.1	.....	7	16	31	42	32	26	47	80	106
+0.2	.....	2	7	21	18	12	11	24	52	66
+0.3	.....	2	3	13	1	1	5	10	32	38
+0.4	.....	1	2	7	.....	.....	2	5	15	16
+0.5	.....	1	.....	1	.....	.....	2	2	7	4
+0.6	.....	.....	.....	.....	.....	.....	1	1	2	1
+0.7	.....	.....	.....	.....	.....	.....	1	.....	.....	.....
+0.8	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
$\bar{y}$	−0.826	−0.709	−0.556	−0.452	−0.558	−0.318	−0.464	−0.403	−0.304	−0.161

TABLE IV

x	F_0–F_9						G_0–G_9					
	$F(x_1)dx$			$F(x_2)dx$			$F(x_1)dx$			$F(x_2)dx$		
	O	C	O–C	O	C	O–C	O	C	O–C	O	C	O–C
−4.4												
−4.3	.....	.....	.....	.....	.....	.....	0	0.1	− 0.1	.....	.....	.....
−4.2	.....	.....	.....	.....	.....	.....	0	0.1	− 0.1	.....	.....	.....
−4.1	.....	.....	.....	.....	.....	.....	0	0.1	− 0.1	.....	.....	.....
−4.0	.....	.....	.....	.....	.....	.....	0	0.1	− 0.1	.....	.....	.....
−3.9	.....	.....	.....	.....	.....	.....	0	0.1	− 0.1	.....	.....	.....
−3.8	0	0.1	− 0.1	.....	.....	.....	0	0.1	− 0.1	.....	.....	.....
−3.7	0	0.1	− 0.1	.....	.....	.....	0	0.1	− 0.1	.....	.....	.....
−3.6	0	0.1	− 0.1	.....	.....	.....	1	0.1	+ 0.9	.....	.....	.....
−3.5	0	0.1	− 0.1	0	0.1	− 0.1	0	0.2	− 0.2	.....	.....	.....
−3.4	1	0.1	+ 0.9	0	0.1	− 0.1	0	0.2	− 0.2	0	0.1	− 0.1
−3.3	0	0.2	− 0.2	0	0.1	− 0.1	0	0.2	− 0.2	1	0.1	+ 0.9
−3.2	0	0.2	− 0.2	0	0.1	− 0.1	1	0.3	+ 0.7	0	0.1	− 0.1
−3.1	0	0.3	− 0.3	0	0.1	− 0.1	0	0.4	− 0.4	0	0.2	− 0.2
−3.0	1	0.4	+ 0.6	1	0.2	+ 0.8	1	0.5	+ 0.5	0	0.2	− 0.2
−2.9	0	0.5	− 0.5	0	0.2	− 0.2	0	0.6	− 0.6	0	0.3	− 0.3
−2.8	1	0.6	+ 0.4	0	0.3	− 0.3	1	0.8	+ 0.2	1	0.3	+ 0.7
−2.7	0	0.9	− 0.9	0	0.4	− 0.4	1	1.1	− 0.1	0	0.4	− 0.4
−2.6	1	1.2	− 0.2	0	0.4	− 0.4	2	1.4	+ 0.6	1	0.5	+ 0.5
−2.5	1	1.5	− 0.5	1	0.5	+ 0.5	2	1.7	+ 0.3	1	0.6	+ 0.4
−2.4	1	1.7	− 0.7	0	0.6	− 0.6	2	2.2	− 0.2	1	0.8	+ 0.2
−2.3	2	2.2	− 0.2	1	0.9	+ 0.1	3	2.9	+ 0.1	1	1.0	0.0
−2.2	2	2.8	− 0.8	1	1.1	− 0.1	3	3.6	− 0.6	1	1.1	− 0.1
−2.1	3	3.5	− 0.5	1	1.4	− 0.4	4	5.1	− 1.1	2	1.4	+ 0.6
−2.0	3	4.2	− 1.2	1	1.7	− 0.7	6	7.0	− 1.0	2	2.0	0.0
−1.9	4	5.2	− 1.2	1	1.9	− 0.9	7	9.8	− 2.8	3	2.3	+ 0.7
−1.8	5	6.5	− 1.5	2	2.5	− 0.5	9	12.9	− 3.9	3	2.7	+ 0.3
	7	8.1	− 1.1	2	3.0	− 1.0	11	16.1	− 5.1	5	3.7	+ 1.3

−1.7	4	9.9	− 5.9	2	3.9	− 1.9	5	19.5	−14.5	1	4.8	− 3.8
−1.6	11	11.7	− 0.7	9	4.9	+ 4.1	9	21.8	−12.8	8	6.2	+ 1.8
−1.5	6	13.6	− 7.6	4	6.8	− 2.8	9	23.4	−14.4	12	8.5	+ 3.5
−1.4	22	15.6	+ 6.4	9	9.4	− 0.4	17	24.7	− 7.7	8	11.6	− 3.6
−1.3	17	17.7	− 0.7	13	12.5	+ 0.5	24	26.1	− 2.1	11	16.2	− 5.2
−1.2	18	19.9	− 1.9	10	15.9	− 5.9	27	28.1	− 1.1	17	22.4	− 5.4
−1.1	16	22.4	− 6.4	23	19.2	+ 3.8	37	32.1	+ 4.9	42	28.3	+13.7
−1.0	45	25.1	+19.9	23	21.7	+ 1.3	48	38.2	+ 9.8	30	35.0	− 5.0
−0.9	37	27.9	+ 9.1	14	23.2	− 9.2	47	41.7	+ 5.3	41	40.9	+ 0.1
−0.8	33	32.8	+ 0.2	24	24.5	− 0.5	52	45.7	+ 6.3	43	45.4	− 2.4
−0.7	35	39.7	− 4.7	20	26.4	− 6.4	87	49.3	+37.7	51	48.8	+ 2.2
−0.6	38	48.8	−10.8	30	30.0	0.0	57	52.0	+ 5.0	44	51.6	− 7.6
−0.5	66	56.5	+ 9.5	23	35.3	−12.3	61	54.6	+ 6.4	70	56.1	+13.9
−0.4	62	62.7	− 0.7	26	42.6	−16.6	57	56.0	+ 1.0	46	62.2	−16.2
−0.3	72	68.8	+ 3.2	51	51.9	− 0.9	60	56.4	+ 3.6	65	69.2	− 4.2
−0.2	84	76.2	+ 7.8	75	63.0	+12.0	50	54.5	− 4.5	61	67.8	− 6.8
−0.1	80	80.8	− 0.8	55	75.0	−20.0	40	49.5	− 9.5	66	61.6	+ 4.4
0.0	85	78.8	+ 6.2	99	84.4	+14.6	47	41.5	+ 5.5	46	51.2	− 5.2
+0.1	66	68.5	− 2.5	104	85.4	+18.6	37	38.3	− 1.3	46	44.6	+ 1.4
+0.2	40	56.1	−16.1	86	79.7	+ 6.3	33	36.6	− 3.6	41	39.4	+ 1.6
+0.3	42	45.1	− 3.1	86	71.5	+14.5	40	32.8	+ 7.2	46	33.7	+12.3
+0.4	31	32.0	− 1.0	58	63.5	− 5.5	30	29.1	+ 0.9	27	30.8	− 3.8
+0.5	24	21.5	+ 2.5	64	52.7	+ 11.3	23	23.4	− 0.4	31	28.7	+ 2.3
+0.6	13	13.8	− 0.8	39	39.0	0.0	10	18.0	− 8.0	27	28.2	− 1.2
+0.7	8	7.5	+ 0.5	23	20.9	+ 2.1	17	13.6	+ 3.4	35	25.8	+ 9.2
+0.8	6	3.3	+ 2.7	13	10.9	+ 2.1		10.1	− 5.1	14	21.9	− 7.9
+0.9	3	1.3	+ 1.7	6	5.8	+ 0.2	5	7.5	+ 0.5	15	16.9	− 1.9
+1.0	3	0.8	+ 2.2	0	2.5	− 2.5	8	4.7	− 1.7	8	11.6	− 3.6
+1.1	1	0.5	+ 0.5	0	1.1	− 1.1	3	2.3	− 0.3	6	6.9	− 0.9
+1.2	0	0.2	− 0.2	0	0.5	− 0.5	2	0.6	+ 1.4	12	3.7	+ 8.3
+1.3	.....	.....	.....	0	0.2	− 0.2	0	0.1	− 0.1	2	1.6	+ 0.4
+1.4	.....	.....	.....	0	0.1	− 0.1	2	0.0	+ 2.0	2	0.5	+1.5
+1.5	.....	.....	.....	.....	.....	.....	.....	.....	.....	2	0.1	+1.9
+1.6	.....	.....	.....	.....	.....	.....	.....	.....	.....	2	0.0	+ 2.0
+1.7	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....

the absolute magnitudes at which the transition from one distribution to the next is supposed to take place are given at the head of the tables between the columns. The mean values of y_1 and y_2 appear at the bottom of each column and may be used for deriving mean absolute magnitudes as shown in previous publications.

The distributions of

$$x_1 = \log |\tau| + 0.2m ,$$
$$x_2 = \log |v| + 0.2m - \log \sin \lambda$$

are given in Table IV. Both observed and computed values and the differences, O−C, are tabulated.

The distributions of x_1 and x_2 are shown graphically in Figures 1–4, the observed values as points joined by straight lines, the values computed from the distribution of absolute magnitudes and linear velocities by continuous curves.

The peculiar velocities were not used for the determination of that part of the distribution-curve of absolute magnitudes which refers to stars brighter than absolute magnitude -2. The reason for this is that the supergiants have a strong galactic concentration and presumably a marked tendency to move in the galactic plane. The τ-component, being nearly perpendicular to this plane, cannot therefore be compared with the radial velocities of stars near the galactic circle. The large difference between the observed and computed $F(x_1)$ for the G stars and for $x < 0.3$ is due to this cause.

The solution for the G stars showed a tendency toward instability, the least-squares solution giving rather large but uncertain negative values (up to -7) for M near -1. To stabilize the solution it was assumed that the values of $F(M)$ from $M = -0.5$ to -1.6 were strictly zero.

Intervals of two magnitudes were used throughout for $M_2 - M_1$, $M_3 - M_2$, etc., in the equations of conditions, as in previous solutions.

The final distributions of absolute magnitude are given in Table V. They are shown graphically in Figures 5 and 6, where the results from peculiar motions are indicated by plus signs, those from parallactic motions by crosses, and the results from the combined solutions by circles.

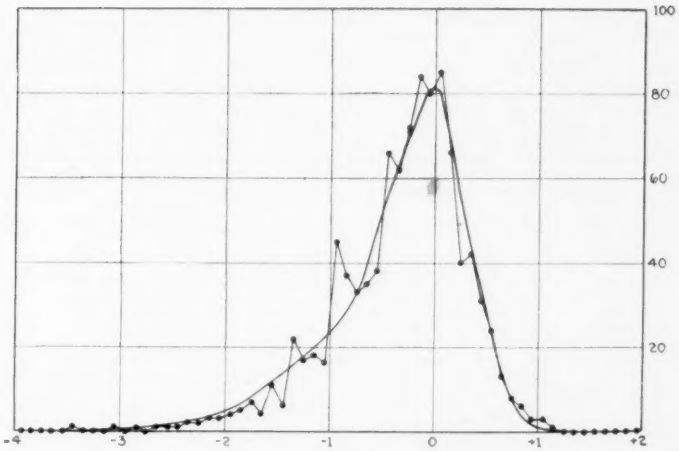

FIG. 1.—Distribution of $x_1 = \log \tau + 0.2\, m$ for stars of spectral types Fo to F9. The dots represent observed numbers within intervals $dx = 0.1$. The smooth curve is computed from the distributions of absolute magnitudes and peculiar radial velocities. For $x < -1.7$ the dots do not indicate independent observations, but show the probable extension of the distribution-curve.

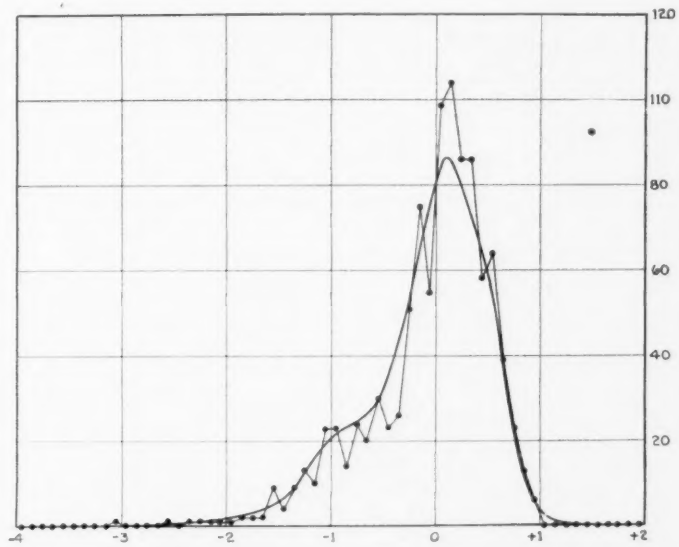

FIG. 2.—Distribution of $x_2 = \log v + 0.2\, m - \log \sin \lambda$ for stars of spectral types Fo to F9. Similar to Fig. 1, but the computed curve is based on a combination of group motions and peculiar radial velocities.

The distribution of absolute magnitudes for the F stars brighter than the sixth apparent magnitude shows three distinct maxima: between absolute magnitudes -5 and -2.0, at $+1.4$, and at $+3.3$. The first group, which includes 9 per cent of the F stars, probably consists of *c* stars (pseudo-Cepheids) and may be called "supergiants." The second group, including 45 per cent of the stars, may be called "normal giants," in conformity with the no-

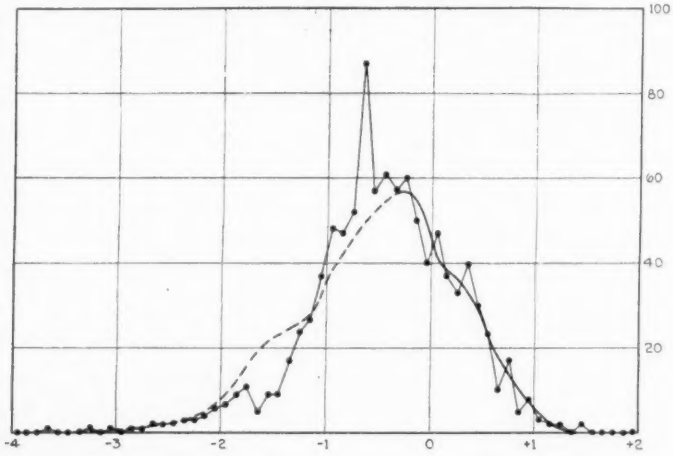

FIG. 3.—Distribution of $x_1 = \log r + 0.2\ m$ for stars of spectral types G0 to G9. Otherwise similar to Fig. 1.

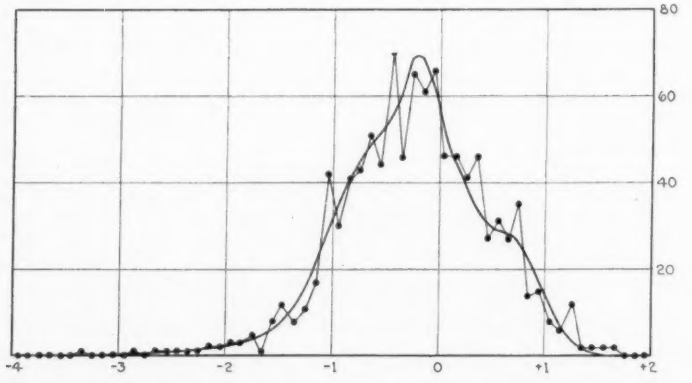

FIG. 4.—Distribution of $x_2 = \log v + 0.2\ m - \log \sin \lambda$ for stars of spectral types G0 to G9. Otherwise similar to Fig. 2.

tation adopted in *Contribution* No. 418,[1] although the stars are somewhat fainter than the normal giants of types G and K. The third group, which includes 46 per cent, may be called "dwarfs," but equally well they may be called "faint giants" and regarded as an extension of the faint-giant group of the G and K stars.

TABLE V

M	$F(M)dM$		M	$F(M)dM$	
	Fo–F9	Go–G9		Fo–F9	Go–G9
−5.4	1	1	0.0	16	78
−5.2	1	2	+0.2	24	118
−5.0	1	4	+0.4	32	120
−4.8	2	5	+0.6	41	85
−4.6	2	6	+0.8	50	48
−4.4	4	8	+1.0	57	22
−4.2	6	10	+1.2	63	8
−4.0	8	12	+1.4	62	5
−3.8	9	13	+1.6	47	5
−3.6	9	14	+1.8	30	13
−3.4	8	14	+2.0	16	19
−3.2	7	13	+2.2	15	23
−3.0	6	12	+2.4	23	25
−2.8	7	13	+2.6	40	26
−2.6	8	15	+2.8	60	27
−2.4	8	16	+3.0	69	26
−2.2	3	16	+3.2	70	24
−2.0	0	11	+3.4	64	20
−1.8	0	2	+3.6	47	18
−1.6	0	0	+3.8	32	18
−1.4	0	0	+4.0	20	18
−1.2	0	0	+4.2	12	17
−1.0	0	0	+4.4	7	15
−0.8	0	0	+4.6	4	12
−0.6	0	0	+4.8	1	6
−0.4	1	1	+5.0	0	3
−0.2	7	10	+5.2	0	2
0.0			+5.4	0	1
			+5.6		

The G stars show three distinct maxima. The supergiants, presumably *c* stars, between $M = -5$ and -1.5 comprise 19 per cent of the stars. The normal giants with a maximum frequency at +0.4 include 49 per cent. The group between $M = +1.6$ and +6 is probably a combination of faint giants, 23 per cent, and real dwarfs, 9 per cent. The reason for assuming the existence of both faint giants

[1] *Astrophysical Journal*, **73**, 40, 1931.

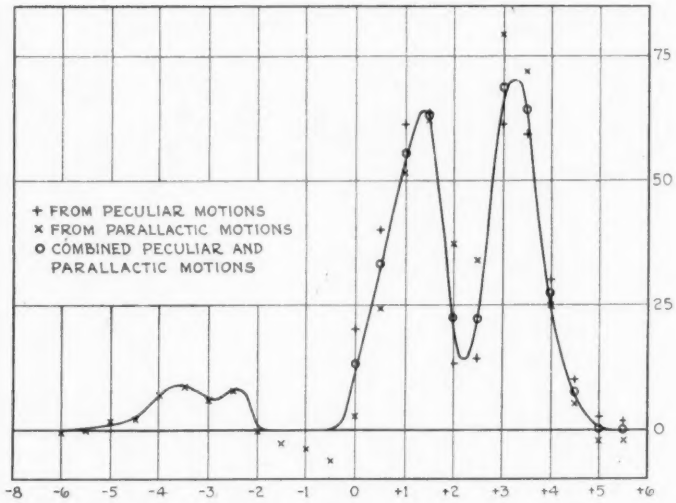
+ FROM PECULIAR MOTIONS
× FROM PARALLACTIC MOTIONS
O COMBINED PECULIAR AND PARALLACTIC MOTIONS
75
50
25
0
-8 -6 -5 -4 -3 -2 -1 0 +1 +2 +3 +4 +5 +6

FIG. 5.—Distribution of absolute magnitudes for stars of spectral types F0 to F9 brighter than the sixth apparent magnitude.

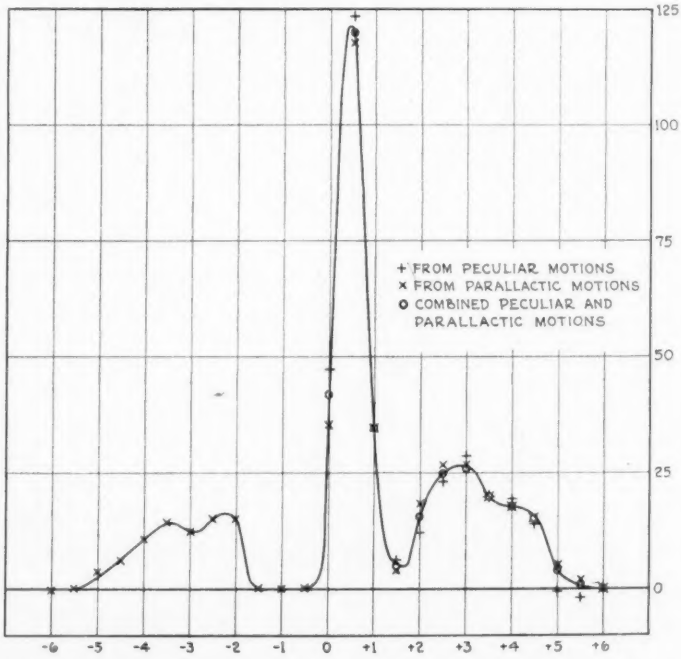

FIG. 6.—Distribution of absolute magnitudes for stars of spectral types G0 to G9 brighter than the sixth apparent magnitude.

and dwarfs is that continuity with the K stars, where the last two groups are well separated, can thus be obtained.

There is some reason to believe that the supergiants also form a double group. Both the F and the G stars give slight indications of two maxima, and since we have evidence among the K stars of a group of bright giants (which are not *c* stars) as distinct from supergiants, the group of bright giants may extend into the region of G and F stars. The large proportion of supergiants as compared with the known number of *c* stars indicates also that the groups here designated as supergiants contain more stars than regular *c* stars.

The complete absence of both F and G stars having absolute magnitudes near $M = -1.0$ is noteworthy. This gap probably exists also among the A stars.

Similar studies of A and B stars will appear in future *Contributions*.

The writer is particularly indebted to Miss Richmond, of the Computing Division, who has done a large part of the numerical work.

CARNEGIE INSTITUTION OF WASHINGTON
MOUNT WILSON OBSERVATORY
April 1931

THE MEASUREMENT OF CORONAL BRIGHTNESS AT THE TOTAL SOLAR ECLIPSES OF MAY 9, 1929, AND OCTOBER 21, 1930

By HARLAN TRUE STETSON, WELD ARNOLD, and JOSEF JOHNSON

ABSTRACT

The Macbeth *illuminometer* as previously used at the Sumatra eclipse of 1926, and described elsewhere, has been *specially adapted for eclipse work* by certain improvements in design. Results in the measured coronal brightness with the improved apparatus were obtained at the May 9 eclipse of 1929 at Alor Star, Malaya, and that of October 21, 1930, at the island of Niuafou.

Light-curves of the entire eclipse are presented, *giving the total illumination at mid-minimum* at the eclipse of 1929 as 0.15 foot candles, and for the eclipse of October 21, 1930, as 0.378 foot candles. The necessity for ultimately correcting all photometric values for the duration of the eclipse is pointed out. It is believed that evidence remains, however, for the increased brightness of the corona at times of sun-spot maximum.

The time of mid-totality is determined from the photometric light-curve with a probable uncertainty of a very few seconds.

During the eclipse of January 24, 1925, many attempts were made to observe the total illumination by means of the Macbeth illuminometer by a number of observers, with more or less discordant results.[1] The improvements in the method of operation and a description of the instrument used the following year in Sumatra by Stetson, Coblentz, and Arnold have been previously published in this *Journal*.[2] Experience gained from manipulation of the instrument in Sumatra led to some alterations in its design to facilitate the settings when the change in illumination becomes most rapid as the eclipse approaches totality.

The instrument makes possible the measurement of values from a few hundredths of a foot candle to more than 10,000 foot candles. It is ordinarily equipped with separate absorbing screens which may be interposed between the source and the comparison prism, on the one hand, or between the reference standard and the comparison prism, on the other hand, as necessity requires; the latter case existing when the unknown source is fainter than the reference standard. The selecting and placing of these separate screens during the rapid changes of illumination considerably delays the observa-

[1] *Transactions of the Illuminating Engineering Society*, **20**, 565, 1925.

[2] **66**, 65, 1927.

tions and leads to possible confusion when every moment is at a premium. Accordingly, with the co-operation of the Leeds-Northrup Company, makers of the instrument, a new piece of apparatus was devised whereby all the required absorbing screens were arranged in sequences on two separate slides that can be manipulated by touch. One of the slides is placed between the source and the comparison prism for use when the intensity of the source is to be reduced. A similar slide containing the needed screens for the comparison lamp is provided for operation between the prism and the reference lamp to be used when the reference lamp is disproportionately bright.

An instrument with these modifications was furnished by the Leeds and Northrup Company for operation at the total eclipse in Norway in 1927. The illuminometer was at that time operated by Stetson and Hendell, in co-operation with the McCormick-Chaloner Expedition at Fagernes under the auspices of the Smithsonian Institution. Heavy cloudiness during the eclipse resulted in no records of scientific value.

The same instrument, after recalibration, was taken on the Harvard Expedition to Alor Star, Malaya, in 1929 with more satisfactory results.[1] As in the case of the 1926 eclipse, it was found convenient to mount the instrument upon two supporting Y's with the diffusing screen so oriented as to be at right angles to the line to the sun during totality. By the use of the diffusing screen the illumination of the corona and its surroundings is integrated so that no appreciable error in illumination results from the change in position of the sun during the series of observations. The instrument was carefully calibrated by means of a reference standard lamp specially prepared for the purpose. Calibrations were carefully checked during practice, and before and after the entire series of eclipse observations. The set-up of the illuminometer and its method of operation are illustrated in Figure 1. Observations of full sunlight and of full moonlight adjacent to eclipse time were made en route by Stetson and Arnold. All the observations during the eclipse were made by Arnold. A summary of selected readings at convenient intervals is given in Table I. The light-curve resulting from all the measures is shown in Figures 2 and 3, the minimum illumination at mid-totality

[1] *Popular Astronomy*, **37**, 370, 1929.

being 0.15 foot candles or a value slightly in excess of that found for the Sumatra eclipse of 1926 (0.138 foot candles). It will be observed that the light-curve appears quite smooth and symmetrical, thus indicating that no rapid changes in sky transparency occurred during the progress of the eclipse although a veil of thin cirrus prevailed in the region of the sun during the entire series of observations.

When we take into consideration the fact that the 1929 eclipse was of considerably longer duration and, therefore on this account,

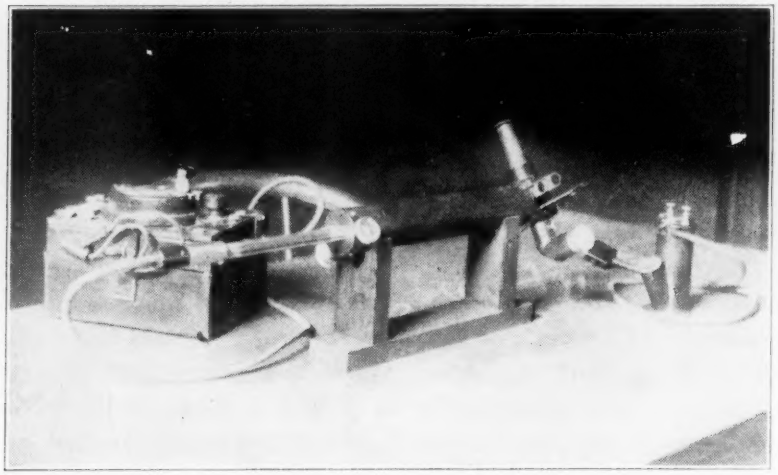

FIG. 1.—The Macbeth illuminometer adapted for eclipse work

more of the inner corona was covered by the moon than in the 1926 event, it appears that the corona in this instance must have been considerably brighter than on the previous occasion. This distinctly favors the hypothesis already suggested of an increase in coronal illumination near the time of maximum solar activity.

The practical and convenient performance of the new absorbing screens left little to be desired in the way of improvements in the apparatus. It was found advisable, however, to have a recorder assist the observer, so that the observer's eye might be exclusively devoted to the matching of light-values. The reading of the illuminometer scale and the recording were efficiently performed by Mrs. Arnold.

In connection with the eclipse expedition of the United States Naval Observatory to Niuafou for observing the total eclipse of October 21, 1930, the illuminometer was taken from Perkins Ob-

TABLE I

SUMMARY OF MEASURES WITH THE ILLUMINOMETER AT ALOR STAR, MALAYA, MAY 9, 1929

Watch Time			Foot Candles	Watch Time			Foot Candles
Hr.	Min.	Sec.		Hr.	Min.	Sec.	
5.....	32	15	6,839	6.....	37	45	0.185
	19	0	4,251		37	51	.203
	34	22	3,312		37	58	.203
	50	1	3,084		38	5	.203
					38	22	.246
6.....	1	10	2,870		38	27	.234
	5	17	3,245		38	35	.272
	8	15	2,280		38	52	.391
	11	7	1,985		39	10	.529
	18	5	1,018		39	33	0.874
	22	37	686		39	53	1.84
	25	19	575		40	8	5.20
	28	43	355		40	34	13.5
	29	54	263		40	44	21.0
	30	53	203		41	36	51.7
	31	19	169		42	42	113
	32	14	101		44	5	172
	33	8	53.4		44	27	201
	34	4	20.0		44	57	226
	34	14	13.5		45	49	263
	34	37	4.1		46	45	361
	34	48	1.6		47	4	357
	35	15	0.391		48	49	501
	35	29	.311		50	5	542
	35	38	.276		51	52	706
	35	48	.288		54	21	862
	36	41	.166		55	54	977
	36	48	.180				
	36	55	.160	7.....	0	17	1,190
	37	7	.180		3	35	1,453
	37	15	.191		7	0	2,065
	37	23	.180		16	36	3,554
	37	31	.203		26	41	5,216
	37	39	0.197		35	26	6,973

servatory by Mr. Josef Johnson, research assistant, and set up in accordance with the practice already outlined. While again the sky was not perfectly clear, satisfactory readings were made by Mr. Johnson during the eclipse, with Mrs. Arnold recording. The resulting light-curve is depicted in Figures 4 and 5. The curve is based

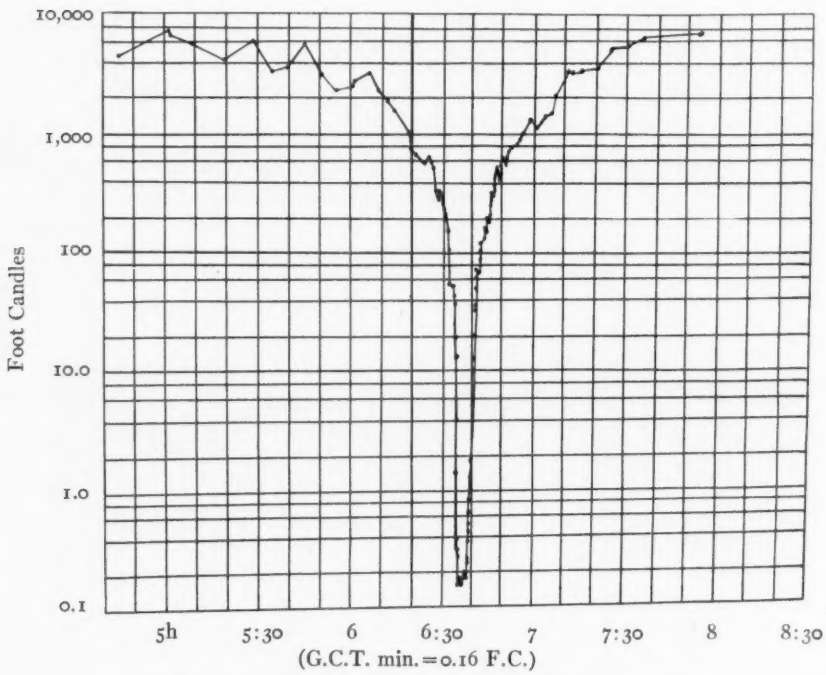

FIG. 2.—Illumination during eclipse of May 9, 1929

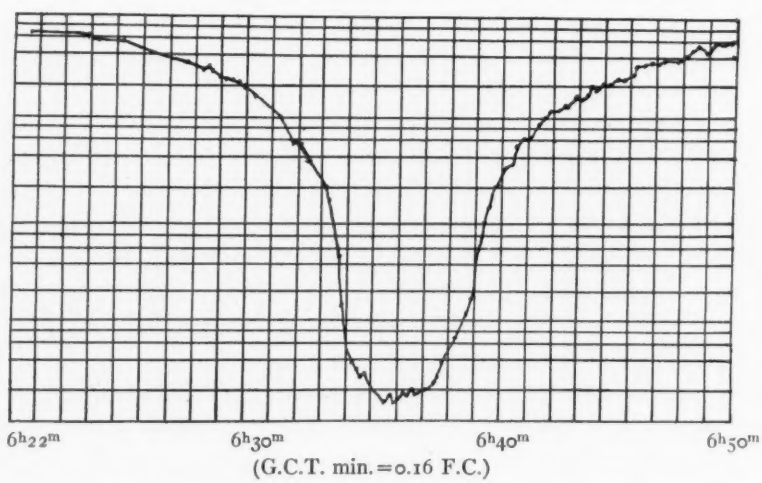

FIG. 3.—Midpart of light-curve of eclipse of May 9, 1929

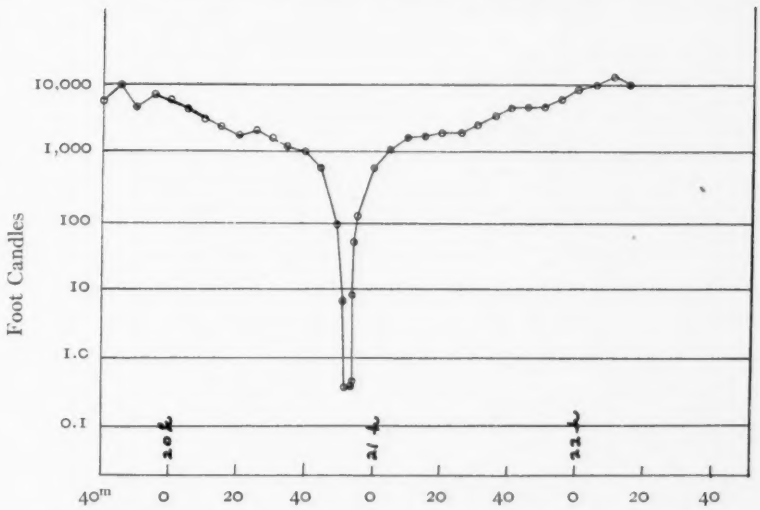

FIG. 4.—Total normal illumination dating solar eclipse, Niuafou, October 21, 1930. (Macbeth illuminometer.) Min. light = 0.378 F.C.

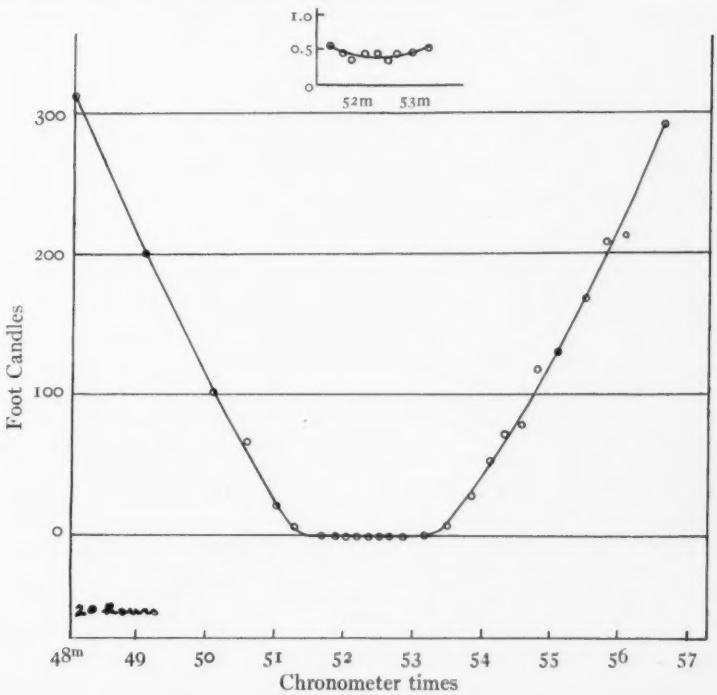

FIG. 5.—Midpart of light-curve through totality, Niuafou, October 21, 1930. (Chronometer 33.2 sec. fast on G.C.T.)

TABLE II

ILLUMINOMETER READINGS AND REDUCTIONS TO FOOT CANDLES
NIUAFOU ECLIPSE, OCTOBER 21, 1930
(Watch 33.2 sec. fast on G.C.T.)

WATCH TIME			SCALE	FILTERS		FOOT CANDLES
Hr.	Min.	Sec.		*A*	*B*	
19	40	3	4.35	4	1	5,830
	45	3	7.55			10,100
	50	3	3.70			4,960
	55	2	5.91			7,910
20	0	2	5.09			6,820
	5	3	3.71			4,970
	10	2	2.81			3,760
	15	2	2.43			3,260
	20	2	1.51			2,020
	25	2	1.81			2,430
	30	2	1.58			2,120
	35	2	20.5	4	2	1,670
	40	2	17.0			1,400
	41	1	16.4			1,350
	42	2	11.8			969
	43	1	8.50			698
	44	3	10.3			846
	45	2	8.00			657
	46	3	6.83			561
	47	2	4.05			332
	48	2	3.78			311
	49	2	2.44			201
	50	3	9.20	4	3	101
	50	32	6.21			68.3
	51	0	2.13			23.4
	51	19	6.43	4	4	6.43
	51	40	4.65	3	4	0.535
	51	49	3.80			.437
	51	59	3.29			.378
	52	9	3.71			.427
	52	21	3.80			.436
	52	30	3.30			.379
	52	38	3.80			.436
	52	49	3.90			.448
	53	9	4.60			0.528
	53	27	7.00	4	4	7.00
20	53	51	2.81	4	3	30.9
	54	5	5.00			55.0
	54	18	6.80			74.8
	54	33	7.50			82.5
	54	46	11.0			121
	55	4	1.61	4	2	132
	55	26	2.10			172
	55	46	2.60			213
	56	2	2.65			217
	56	33	3.61			297
	57	1	3.14			257
	57	32	4.80			394

TABLE II—*Continued*

Watch Time			Scale	Filters		Foot Candles
Hr.	Min.	Sec.		*A*	*B*	
20	58	2	5.19			426
	59	6	7.00			575
21	0	3	8.90			731
	1	2	9.50			780
	2	2	10.1			829
	3	2	13.5			1,110
	4	3	16.5			1,350
	5	3	14.5			1,190
	10	1	1.61	4	1	2,150
	15	2	1.63			2,180
	20	3	1.70			2,280
	25	12	1.62			2,170
	30	3	2.15			2,880
	35	4	2.94			3,940
	40	1	3.80			5,080
	45	13	3.69			4,930
	50	2	3.75			5,020
	55	2	5.30			7,100
22	0	3	7.25			9,700
	5	1	7.80			10,400
	10	3	8.20			11,000
	15	1	7.60			10,200

on seventy measurements, nine of which were obtained during totality. The observations were made at five-minute intervals during the larger partial phases, the intervals being decreased to one minute and then to thirty seconds during the smaller partial phases. As many readings as possible were made during totality. The times, readings, and equivalent foot candles are tabulated in Table II.

The times were recorded by means of a Ditisheim pocket chronometer which, on comparison with radio time signals, was found to be 33.2 seconds fast on G.C.T.

The instrument was carefully restandardized by Stetson and Johnson upon return to the Perkins Observatory. The value for minimum intensity, 0.378 foot candles, is surprisingly large, being over twice that obtained at the Sumatra eclipse of January 14, 1926. The relatively shorter duration of the eclipse, however, as compared with the eclipses of 1926 and 1929, would make for increased illumination on account of a larger proportion of the inner corona being uncovered. It is entirely possible, however, that a considerable amount of

haze and cirrus cloud during totality increased the brightness of the surrounding sky. As the illuminometer with its diffusing screen covers a considerable area in the sky, readings with the instrument comprise the illumination of both corona and sky, and not the corona alone. As the extent of the corona, however, is always open to question, and the general sky illumination is dependent upon the conditions of the eclipse, it has not been deemed wise to circumscribe the area of operation in the measurement of brightness with the illuminometer. Indications, however, are that the eclipse was of more than usual brightness, again in keeping with the hypothesis that the corona is brightest during periods of maximum solar activity.

When a sufficiently extended series of photometric observations with the illuminometer have been acquired, it is hoped that it may be possible to apply corrections which must depend upon the relative distances of the sun and moon during the total phase, so that more representative values of the brightness of the corona itself may be otained for intercomparison.

It is interesting to note that the time of mid-totality deduced from the form of the light-curve is but a few seconds in error from the predicted time of mid-eclipse.

PERKINS OBSERVATORY
OHIO WESLEYAN UNIVERSITY
March 1931

THE POTSDAM SCALE OF VISUAL MAGNITUDES[1]

By FREDERICK H. SEARES

ABSTRACT

Potsdam measures of the Polar Sequence.—Visual magnitudes of 36 stars of the Polar Sequence brighter than about 12.3 occur in the following Potsdam catalogues: PD, 11 stars; PDP (*Potsdam Polar Catalogue*, by Hassenstein), 22 stars; PDP supplement (PPS), 26 stars (Table III).

Comparison with International Scale.—About 100 stars appearing in both the PD and the PDP and numerous measures of the same star by the different observers afford data for the reduction to a homogeneous system of the three overlapping series of PS observations. The color system adopted for this reduction, which is that of Müller for the years 1909–1916, agrees with the Mount Wilson photovisual system. The reductions to the international zero-point for the three series are consistent (Table IV); the mean scale differences MW Pv−P Vis (Table V) are small and show no important systematic change within the interval 2–12 mags.

Relation of the PD and PDP to the MW Pv scale.—The discussion of the PS stars leads finally to the relations between the scales of the PD and PDP and the MW Pv scale expressed by equations (15) and (16). These formulae are subject to certain modifications and restrictions which are explained in the text.

The *Potsdam Polar Durchmusterung* (PDP),[2] aside from its value as a collection of magnitudes, is a substantial contribution to our knowledge of the visual scale of the Polar Sequence, and, through stars common to it and the *Potsdam Generalkatalog* (PD),[3] provides means of determining the relation of the PD to the Mount Wilson photovisual system with greater accuracy than has hitherto been possible.

The PDP gives careful measures with a Zöllner photometer of the B.D. stars of magnitudes 7.5–9.5 in declinations +80° to +90° and of selected stars within these limits of brightness down to +73°. It also includes 22 stars of the Polar Sequence (PS) between magnitudes 6.4 and 10.8, each observed four times, generally by Müller. Further, a supplementary list[4] gives results for 26 stars of the Sequence, magnitudes 7.2–12.3 (PPS), which were extensively measured in special

[1] *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 431.

[2] *Publikationen des Astrophysikalischen Observatoriums zu Potsdam*, **26**, Part 2 (No. 85), 1927. Observations by G. Müller, E. Kron, and A. Kohlschütter; reductions and discussion by W. Hassenstein.

[3] *Ibid.*, **17**, 1907.

[4] *Op. cit.*, p. 141.

zones by both Müller and Kron, who made, in general, ten and nine observations, respectively, on each star. All these measures are referred to primary and secondary standards of the eighth and ninth magnitudes, which in turn are based on 8 seventh-magnitude fundamental stars in declination $+80°$ used for the PD. Finally, the PD itself includes the brighter stars of the Polar Sequence, measured once each by both Müller and P. Kempf as a part of the regular program for that catalogue. Altogether these three overlapping series of observations of the Sequence cover a range of 10 magnitudes, as shown by columns 3, 4, and 5 of Table III.

Hassenstein (p. 30 of his paper) has already compared the PPS results with the Harvard and Mount Wilson scales. It is desirable, however, to extend this comparison to the other series of measures and allow, at the same time, for the influence of color equation, which is not negligible. When this has been done, the relation of both the PDP and the PD to the Mount Wilson photovisual system can easily be determined.

COLOR EQUATIONS

The first question is the relative color equations of the observers—Müller (M), Kempf (K), Kron (Kr), and Kohlschütter (Ko). Müller's measures may be divided as follows:

M	Observations with K for PD	1886–1905
M_1	Observations with Kr for PDP	1909–1916
M_2	Observations with Ko for PDP	1918–1920

1. *M and K.*—Müller and Kempf have compared their observations for the four sections of the PD with results for the differences M − K (unit = 0.01 mag.) as given in Table I.

The original data show that the color coefficient is independent of magnitude; Table I indicates that its value throughout the nineteen-year interval is sensibly constant and equal to +0.04. The zero-point difference, which is the value of M − K for W stars, gradually changes from 0.00 to −0.05. Since the present discussion is concerned mostly with the polar region, the required equation is

$$M-K=-0.05+0.04\,C\,. \qquad (1)$$

Especially valuable for the discussion are the magnitudes of 128 stars, mostly between 6.0 and 8.0, observed for both the PD and

PDP. Measures by M_1, M_2, Kr, and Ko give the differences collected in Table II, in which figures in parentheses indicate numbers of stars. The MW color-indices which correspond to the HD spectra are from *Contribution* No. 415, Table XI.[1]

2. *M_1 and M_2.*—The number of differences available is too small for a satisfactory comparison of Müller's observations of 1909–1916 with those of 1918–1920. The results of Table II are checked, however, by an indirect comparison based on well-determined values of

TABLE I

VALUES OF M−K FOR THE PD

DECLINATION	COLOR				*Potsdam Publikationen*	OBSERVATION INTERVAL
	W	GW	WG	G		
0° to +20°	0	.....	.....	+4	9, 485	Oct. 1886—Nov. 1892
+20 to +40	−3	−3	−1	+3	13, 450	Sept. 1890—May 1898
+40 to +60	−4	−3	0	+3	14, 436	Oct. 1897—Dec. 1902
+60 to +90	−5	−6	−1	+1	16, 261	Oct. 1901—Aug. 1905
MW color-index	+0.02	+0.30	+1.04	+1.66	.....	.....

$\frac{1}{2}(M_1+Kr)-M_2$ given by Hassenstein (p. 14). These differences are independent of the color of the stars, but vary with the magnitude, and for the interval 6–8 have the mean value −0.06. Hence

$$M_1-M_2=\tfrac{1}{2}(M_1-Kr)-0.06\ ,$$

and, with values of M_1-Kr from Table II, we find

	A1	F1	G2	Ko	K5
$M_1-M_2=$	−5	−5	-8^5	-6^5	-4^5

These differences are approximately the same as those given in Table II. In neither case is there any evidence of a change in Müller's perception of color. The only difference is one of scale, which, for the interval in question, amounts to 0.05 or 0.06 mag. In preparing the PDP catalogue M_2 was reduced to the system $\frac{1}{2}(M_1+Kr)$, which was adopted as the standard.[2]

[1] *Astrophysical Journal*, **72**, 338, 1930.

[2] Since M_2 has thus been corrected for scale divergence, it is surprising to find that the differences M_1-M_2 in Table II, which were derived from catalogue data, do not have a

3. *M_1 and Ko.*—This case is in all respects similar to that preceding; the color systems of Ko and M_1 may be regarded as the same. The mean difference M_1-Ko from Table II (+0.14 for mag. 7.5) is also in substantial agreement with the scale divergence found by Hassenstein. Since the divergence from $\frac{1}{2}(M_1+Kr)$ is in general the same for both M_2 and Ko, an instrumental origin is suggested. This circumstance led to the reduction of both M_2 and Ko to $\frac{1}{2}(M_1+Kr)$ as the adopted system for the PDP.

TABLE II

DIFFERENCES FROM STARS COMMON TO PD AND PDP

COMPARISON	HARVARD SPECTRUM				
	A1	F1	G2	Ko	K5
PDP−PD	− 5(24)	− 6(25)	0(22)	+ 1(28)	+7(13)
M_1-Kr	+ 2.5(35)	+ 2(27)	−5(23)	− 1(34)	+3(16)
M_1-M	− 2(39)	− 5(37)	−3(25)†	+ 2(34)	+8(17)
M_1-M_2	− 5(6)*	− 9(8)	−5(6)‡	− 2(10)	.....
M_1-Ko	+24(2)	+14(5)	0(6)†	+20(9)	.....
MW color-index	−0.05	0.11	0.49	0.91	1.22

* Sp. A4, C.I. = −0.01.
† Sp. G1, C.I. = +0.43.
‡ Sp. Go, C.I. = +0.39.

4. *M_1 and Kr.*—Aside from the K5 stars, the differences in Table II indicate a small negative color coefficient. This agrees with the results of an extensive comparison by Hassenstein (p. 12), which, in the mean, are represented by

$$M_1-Kr=+0.04-0.03\,C\,. \qquad (2)$$

For magnitude 7.5 the first term (scale difference for white stars) would be +0.02. With this modification, the formula fits the differences in Table II reasonably well. Other series of measures, however, give a

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mean value of zero. The explanation lies in the fact that in reducing M_2 to $\frac{1}{2}(M_1+Kr)$ Hassenstein (p. 15) used the mean scale divergence for M_2 and Ko, and thus obtained a zero correction for stars brighter than 9.2. Although closely accordant for the fainter stars, the scales of M_2 and Ko are not quite the same for bright stars. That the slender data of Table II give the same mean value of M_1-M_2 as the hundred or so stars used for the indirect comparison testifies to the consistency of Müller's measures.

different result. For example, the special PPS zones (Hassenstein, p. 141), which belong with the M_1 group of Müller's observations, include 26 stars of the Polar Sequence that were extensively measured by both Müller and Kron. For these the color coefficient is consistently positive and, in the mean, equal to +0.10. The differences M_1-Kr are given for each star in column 6 of Table III. The individual values of the scale difference and color coefficient for the several groups of stars are:

Group	IV	V	VI	VII	VIII	IX	Mean
Scale diff.	−0.01	+0.09	+0.06	−0.10	−0.08	−0.12	−0.02
Color coeff.	+0.09	+0.02	+0.10	+0.23	+0.10	+0.05	+0.10

Although the number of stars in each group is small, the results are of considerable weight because of the numerous measures. That the mean color coefficient for these zones is really positive can scarcely be doubted. The negative sign in the scale differences for groups VII, VIII, and IX indicates a small scale divergence between the two observers. For the discussion connected with Table III the mean scale has been used, but Kron's measures of the PPS were first reduced to the color system of Müller. The adopted relation is

$$M_1-Kr=0.00+0.10\,C\,. \qquad (3)$$

The disagreement between this expression and formula (2) (or the differences of Table II) indicates that the relative color perception of the two observers did not remain constant. The change which must have taken place doubtless could be studied more in detail by collecting chronologically all the data for M_1 and Kr given in the PDP; but since Dr. Hassenstein has kindly written me that it would probably be impossible to determine whether the change occurred in Müller or in Kron, or perhaps in both, the considerable labor involved seems scarcely justified.

5. *M_1 and M.*—The constancy of the relative color coefficient for Müller and Kempf throughout the interval 1886–1905 (Table I) makes it probable that the color perception of these two observers did not change. The comparison of Müller's observations in 1918–1920 with those of 1909–1916 (M_2 and M_1) also gives little or no evi-

dence of change. For the interval 1909–1916 as against 1901–1905, we have the differences $M_1 - M$ in Table II. The dependence on color is approximately of the form

$$M_1 - M = -0.04 + 0.09\,C\,. \qquad (4)$$

6. *PDP and PD.*—The differences in the first line of Table II, depending on the mean magnitudes of 112 stars appearing in both catalogues, are of the form

$$PDP - PD = -0.06 + 0.10\,C\,. \qquad (5)$$

Sixteen additional stars also occur in both catalogues; but since the measures for the PDP are exclusively by Müller, they are not, strictly speaking, on the system of the PDP, and on that account were not included.

The values of PDP − PD may be used to test the consistency of some of the other results in Table II. Since

$$PDP = \tfrac{1}{2}(M_1 + Kr) \text{ and } PD = \tfrac{1}{2}(M + K)\,, \qquad (6)$$

we have

$$PDP - PD = (M_1 - M) - \tfrac{1}{2}(M_1 - Kr) + \tfrac{1}{2}(M - K)\,. \qquad (7)$$

The agreement of the direct comparison with that obtained by combining the differences on the right of this formula is as follows:

	A1	F1	G2	K0	K5
Direct	−0.05	−0.06	0.00	+0.01	+0.07
Indirect	− .05[5]	− .08	− .02	+ .02	.06[5]
Difference	+0.00[5]	+0.02	+0.02	−0.01	+0.00[5]

Had the various differences involved been derived from exactly the same stars, observed in the same manner by all the different observers, the direct and indirect comparisons would have checked exactly. As it is, the values of M − K depend on all the stars in the 60°–90° section of the PD; further, the PDP includes measures by Kohlschütter and by Müller during 1918–1920 (M_2). These have been reduced to the system $\frac{1}{2}(M_1 + Kr)$, but by means of relations based on all the available material in the PDP. The agreement is therefore satisfactory.

Although formula (5) depends on a small number of stars, it is probably applicable to all stars brighter than the eighth magnitude. Whether it can be used to reduce the fainter stars in the PDP to the system of the PD is of course open to some question; for the present, however, we shall assume that such is the case.

Although we are here concerned chiefly with color equations, it may be remarked that Hassenstein (p. 23) finds the scales of the PD and the PDP to be sensibly parallel (magnitudes 6.1–8.3).

POTSDAM VISUAL MAGNITUDES OF THE POLAR SEQUENCE

The reduction of the Potsdam measures to a homogeneous system and their comparison with the Mount Wilson results are shown in Table III. The numbers prefixed to the following paragraphs correspond to those of the column headings in the table.

1. For convenience in forming means, the 36 stars measured at Potsdam are divided into nine groups, each including both white and red stars and covering a small interval in brightness. Because of the 2-mag. interval separating it from the next fainter star, Polaris (1s) is necessarily considered by itself.

2. The star numbers are those of Pickering.

3. The magnitudes are from the PD, but have been reduced to the system M_1. Since

$$M_1 - PD = (M_1 - M) + \tfrac{1}{2}(M - K) ,$$

the data of Table II give the corrections

$$\left.\begin{array}{lccccc} \text{MW color-index} & -0.05 & +0.11 & +0.43 & +0.91 & +1.22 \\ M_1 - PD & -0.04^5 & -0.07 & -0.04 & +0.01^5 & +0.08 \end{array}\right\} \quad (8)$$

The only exception is Polaris, which was observed by Müller alone. For this the color correction $(M_1 - M)$ is zero. The catalogue magnitude of 2.31 has been changed, however, by −0.14, in accordance with the results of an intricate discussion by E. Zinner,[1] who has shown that a few of the brightest stars of the PD require small corrections to eliminate an obscure systematic error. The number of observations is usually two for each star (numerous for 1s; four each for stars 1 and 3).

[1] *Bamberg Veröffentlichungen*, **2**, 78, 1926.

4. These magnitudes are taken from the main catalogue of the PDP, mostly without change. The number of observations is four

TABLE III

COMPARISON OF POTSDAM VISUAL WITH MOUNT WILSON PHOTOVISUAL MAGNITUDES

Group	Star	PD	PDP	PPS	M_1-Kr		Mean P	MW		Pv−P	Pv_1-P	Pv_2-P
					Direct	Corr.		Pv	C.I.			
1	2	3	4	5	6	7	8	9	10	11	12	13
I	1s	2.17	.....	.....	.....	.....	2.01	2.08	0.49	+ 7	+ 3	.....
II	1	4.63	.....	.....	.....	.....	4.47	4.37	0.01	−10	− 6	.....
	1r	5.21	.....	.....	.....	.....	5.05	5.09	1.57	+ 4	+10	.....
	2	5.33	.....	.....	.....	.....	5.17	5.28	−0.11	+11	+25	.....
	3	5.74	.....	.....	.....	.....	5.58	5.56	0.18	− 2	+ 6	.....
	4	6.04	.....	.....	.....	.....	5.88	5.84	0.14	− 4	−18	.....
III	2s	6.43	6.36	.....	.....	.....	6.24	6.30	0.17	+ 6	+ 5	.....
	3s	6.42	6.40	.....	.....	.....	6.27	6.35	0.26	+ 8	+14	.....
	2r	6.50	6.41	.....	.....	.....	6.35	6.32	1.58	− 3	+ 3	.....
	5	6.65	6.66	.....	.....	.....	6.50	6.45	0.01	− 5	− 6	.....
IV	6	7.23	7.20	7.23	− 3	− 4	7.08	7.06	0.06	− 2	− 1	.....
	3r	.....	7.58	7.56	+ 9	− 5	7.46	7.57	1.41	+11	+ 7	.....
	7	7.70	7.63	7.56	− 6	− 4	7.46	7.55	−0.17	+ 9	+ 8	.....
	8	.....	8.24	8.09	+ 2	0	8.01	8.13	0.25	+12	0	.....
	4r	.....	8.24	8.39	+12	+ 2	8.22	8.27	1.00	+ 5	−14	.....
V	5r	.....	8.74	8.78	+11	− 4	8.66	8.63	1.53	− 3	.....	.....
	9	.....	9.16	8.92	+10	+ 8	8.87	8.83	0.17	− 4	−18	.....
	10	.....	9.14	9.08	+ 9	+ 8	8.98	9.06	0.12	+ 8	− 6	+ 2
	6r	.....	9.32	9.37	+13	0	9.24	9.24	1.26	0	+ 5	+ 9
VI	11	.....	9.77	9.86	− 1	− 3	9.70	9.56	0.24	−14	−19	−14
	4s	.....	9.96	9.98	+12	+ 7	9.85	9.83	0.50	− 2	− 6	− 1
	12	.....	9.98	10.00	+17	+14	9.86	9.77	0.31	− 9	− 9	− 8
	7r	.....	9.98	10.09	+12	+ 1	9.92	9.87	1.11	− 5	− 9	− 7
	5s	.....	10.12	10.12	+22	+11	10.00	10.06	1.06	+ 6	− 3	− 3
VII	13	.....	10.43	10.54	−10	−12	10.37	10.37	0.22	0	−14	−11
	14	.....	10.37	10.43	+ 4	0	10.28	10.56	0.42	(+28)	+16	+18
	8r	.....	10.57	10.70	+22	+12	10.54	10.46	0.98	− 8	− 7	− 6
	6s	.....	10.82	10.82	− 2	− 9	10.70	10.72	0.68	+ 2	− 5	− 4
VIII	15	.....	.....	10.99	− 6	−10	10.86	10.88	0.38	+ 2	+ 4	+ 4
	9r	.....	.....	11.14	0	− 9	11.01	.....	.....	.....	.....	.....
	16	.....	.....	11.39	− 6	−10	11.26	11.22	0.36	− 4	− 3	− 4
	17	.....	.....	11.47	− 2	− 7	11.34	11.30	0.54	− 4	+ 4	− 6
IX	18	.....	.....	11.98	− 9	−13	11.85	11.90	0.39	+ 5	+ 3	+ 1
	10r	.....	.....	12.10	−10	−16	11.97	12.03	0.58	+ 6	.....	+ 6
	11r	.....	.....	12.32	− 7	−18	12.19	12.07	1.12	−12	− 7	−19
	19	.....	.....	12.30	−11	−17	12.17	12.24	0.44	+ 7	+ 7	+ 6

for each star, all by Müller (M_1), except a single observation by Kron on each of six stars. Kron's measures were reduced to the system M_1 by means of

$$M_1-Kr=+0.08-0.03C\,.$$

The first term was derived from the uncorrected magnitudes; the color coefficient is from equation (2).

5. The PPS magnitudes from the Polar Sequence zones (Hassenstein, p. 141) are the means, in general, of ten observations on each star by Müller and nine by Kron. Kron's measures have been reduced to the system M_I by equation (3). The direct means, uncorrected for color, are the data used by Hassenstein for his comparison with Mount Wilson and Harvard results. Evidence of the internal consistency of the Potsdam measures is afforded by the close agreement of these magnitudes with the independent series in column 4. The scales are very nearly parallel. The only point requiring comment is the appearance of an appreciable color equation, hardly to be expected since both series are supposedly on the color system M_I. The discrepancy suggests that at least part of the difference in the color coefficients of equations (2) and (3) is due to Müller.

6. Differences between the mean magnitudes for M_I and Kr, the latter uncorrected for color, as given in the PPS. Graphical solutions based on these differences give the results shown on page 135, which, in the mean, are expressed by formula (3).

7. The values of $M_I - Kr$ after correction of Kr for color by equation (3). They represent the combined influence of accidental error and scale difference between the two observers for the PPS zones. Except for a slight divergence affecting the faint stars, the agreement is excellent.

8. The weighted mean Potsdam magnitude, formed from the data in columns 3, 4, and 5 after reduction to the zero-point of the Mount Wilson photovisual system (ninth column). As they stand, the magnitudes in these three columns are supposed to refer to the color system and zero-point of M_I and, in consequence, may show a color equation relative to the photovisual magnitudes. If so, this circumstance must be taken into account in determining the zero-point correction for the three series of measures. As a matter of fact, neither the PD nor the PPS series shows any deviation from the Pv color system, and all the magnitudes, for red stars as well as white, may be used unchanged for the derivation of the zero-point reduction. The results for individual groups of stars and the means for the

series are in the third and fifth columns of Table IV. In the case of the PDP, however, the color difference between columns 4 and 5 of Table III, which was referred to in paragraph 5, reappears when the comparison with Pv is made. We thus find

$$\text{Pv}-\text{PDP}=-0.15+0.06\,C\,. \qquad (9)$$

The zero-point reduction found with the aid of this equation is shown in the fourth column of Table IV.

TABLE IV

ZERO-POINT DIFFERENCES MW Pv—POTSDAM VISUAL

Group	Mean Pv	PD	PDP	PPS
I	2.1	(−0.09)	.....	.....
II	5.2	.16	.....	.....
III	6.3	.15	−0.13	.....
IV	7.7	−0.16	.09	−0.05
V	8.9	.....	.20	.10
VI	9.8	.....	.18	.19
VII	10.5	.....	−0.13	.17
VIII	11.1	.....	.....	.15
IX	12.1	.....	.....	−0.15
Wtd. mean	.....	−0.16	−0.15	−0.13

In combining the differences to form Table IV, the values for star 14, which are discordant by about 0.3 mag., were rejected. Were the data wholly consistent and free from observational error, the three mean values of the zero-point reduction would be the same. The agreement is quite satisfactory.

Since the color coefficient in formula (9) also appears when the PDP magnitudes are compared with those of the PD and PPS, it must be retained in forming the weighted mean given in column 8, if the results are to be homogeneous. The numbers of observations for the three Potsdam series are usually 2, 4, and 19, respectively; the weights adopted in forming the means are 1, 2, and 4.

9. The MW photovisual magnitudes are those given in *Transactions of the International Astronomical Union*, **1**, 71, 1922.

10. The color-indices are from the same source, and are the means of C_m and C_e.

11. The residuals MW Pv−P, formed by subtracting the values in column 8 from those in column 9. They represent the combined

effect of accidental error and residual scale error between the Mount Wilson photovisual and the Potsdam visual magnitudes, the latter reduced to a homogeneous color system agreeing with that of the photovisual system. The discussion of these differences for the different groups of stars confirms this agreement. The effects of accidental error are partially eliminated by forming the mean difference for each group. These means are in the third column of Table V.

TABLE V

MEAN DIFFERENCES IN SCALE

Group	Mean Pv	Pv−P	Pv_1−P	Pv_2−P	Mean
I	2.1	+0.07	+0.03	.....	+0.05
II	5.2	.00	+ .03	.....	+ .01
III	6.3	+ .01⁵	+ .04	.....	+ .03
IV	7.7	+ .07	.00	.....	+ .03⁵
V	8.9	.00	− .06	+0.05	.00
VI	9.8	− .05	− .09	− .07	− .07
VII	10.5	− .02	− .02⁵	− .01	− .02
VIII	11.1	− .02	+ .02	− .02	− .01
IX	12.1	+0.01⁵	+0.01	−0.02	0.00

12. The residuals in this column are analogous to those in column 11, but represent a comparison with photographic magnitudes reduced to the photovisual system by subtracting the color-indices of the individual stars. The magnitudes used are the international values,[1] while the color-indices are those derived at Mount Wilson by the method of exposure ratios.[2] The resulting photovisual magnitudes are wholly independent of those in column 9. The mean differences for groups of stars are in the fourth column of Table V.

13. Differences obtained as in the case of column 12 except that the color-indices are those derived by Malmquist and Baade.[3] Mean results are in the fifth column of Table V.

The agreement of magnitude scales shown by the comparisons summarized in Table V is very satisfactory. A small systematic difference is perceptible but not of serious importance. Only one mean difference in the last column of Table V requires comment. The persistence of the negative sign in the three series of residuals for the

[1] *Transactions of the International Astronomical Union*, **1**, 71, 1922.

[2] *Ibid.*

[3] *Meddelande från Lunds Astronomiska Observatorium*, Ser. II, No. 37, p. 21, 1927.

stars of group VI suggests a small systematic error in the Potsdam measures near the tenth magnitude. The differences for star 14 (group VII) in columns 12 and 13 of Table III are still rather large, but more satisfactory than the residual in column 11. This residual is probably to be explained by an appreciable error in the Mount Wilson photovisual magnitude, or possibly by a slight variability in the star.

RELATION OF THE PDP AND THE PD TO THE MOUNT WILSON PHOTOVISUAL SYSTEM

The homogeneous system to which the Potsdam magnitudes have been reduced is not that of either the PDP or the PD, but that defined by Müller's measures of the PPS (M_I), which happens to agree with that of the photovisual system. The difference in color perception revealed by Müller's measures in columns 4 and 5 of Table III raises a question, however, as to whether the adopted system is really representative of M_I. The assumption that such is the case is supported by the following facts: (*a*) Müller's PPS measures are much more numerous than his PDP observations of Polar Sequence stars. (*b*) They agree in color perception with the PD stars when these are reduced to the system M_I. (*c*) They lead to a color coefficient for the PD which agrees well with that found indirectly by an entirely different method. On the other hand, the relative color perception of Müller and Kron for the PPS, equation (3), is not that which holds for the PDP, equation (2). The foregoing assumption with respect to Müller therefore carries with it the supposition that Kron's color perception was abnormal during the measurement of the PPS zones. This, however, is possible, for there is evidence that the color sensibility of neither observer remained wholly constant. Nevertheless, it must be admitted that the color coefficients of the PDP and the PD relative to the photovisual system are left somewhat in doubt.

These uncertainties fortunately do not affect the zero-point reductions. This means that the relations between the magnitudes of blue stars are known for the various systems, which in itself is an important gain. On the basis of Tables IV and V we adopt

$$\text{MW Pv} - M_I = -0.15 \qquad (10)$$

and thus assume that the color system of the Potsdam magnitudes in column 8 of Table III is really representative of the system M_1. Equations (6) give

$$M_1 = PDP + \tfrac{1}{2}(M_1 - Kr) , \qquad (11)$$

$$M_1 = PD + (M_1 - M) + \tfrac{1}{2}(M - K) , \qquad (12)$$

whence, substituting into (10), we find

$$MW\ Pv - PDP = -0.15 + \tfrac{1}{2}(M_1 - Kr) , \qquad (13)$$

$$MW\ Pv - PD = -0.15 + (M_1 - M) + \tfrac{1}{2}(M - K) . \qquad (14)$$

With the aid of (1), (2), and (4) these reduce to

$$MW\ Pv - PDP = -0.13 - 0.01\,C , \qquad (15)$$

$$MW\ Pv - PD = -0.22 + 0.11\,C ,^{1} \qquad (16)$$

which may be accepted as the final result of the discussion. The assumption involved in (10) is of course carried over into the color coefficients of (15) and (16). The constant terms in these formulae, on the other hand, are open to little question and represent the corrections which will reduce to the photovisual system the magnitudes of blue stars as given in the PDP and the PD.

It should be noted that the value used for $M_1 - Kr$ in deriving (15) is that of equation (2), found by Hassenstein (p. 12) from a discussion of several hundred stars, and applies to the PDP as a whole. Although this agrees in part with the differences in Table II based on stars common to the PDP and the PD, it must not be overlooked that limited series of observations may require a different correction. For example, we have found that the PPS zones indicate a relative color perception for these observers which is expressed by formula (3).

In the case of formula (16) it is to be remarked that the constant term involves the first term of the difference M − K, equation (1), which depends on the declination (see Table I). That used is satisfactory for stars north of +20°. For 0° to 20° the first term in (1) disappears, with the result that the corresponding constant in (16) becomes −0.20. Further, the differences M − K are practically inde-

[1] As explained below, this equation applies to magnitude 7.5, declinations +20° to +90°. For other magnitudes use the color coefficients given in the eighth column of Table VI, *Mt. Wilson Contr.*, No. 288; *Astrophysical Journal*, **61**, 284, 1925. For declinations 0° to +20° replace the constant term by −0.20.

pendent of magnitude, but it is not certain that this is true for the differences $M_1 - M$. The values used for $M_1 - M$ depend on stars common to the PDP and the PD (6.1–8.3), which have a mean magnitude of about 7.5. The color coefficient in (16) therefore applies to this magnitude, but may not hold for brighter or fainter stars. The value found agrees, however, with that for the same magnitude derived indirectly with the aid of Harvard magnitudes.[1] In this connection it may be remarked that had we supposed Müller's color system to be represented by the PDP magnitudes in Table III, rather than by those of the PPS, equation (10) would have been replaced by (9), and the resulting color coefficient in (16) would have been +0.14, which seems inadmissibly large.

As for the dependence of this coefficient on magnitude, it is probably best for the present to assume that equation (16) applies to magnitude 7.5 and to use for other magnitudes the values of the coefficient given in *Contribution* No. 288.[2] In other respects, however, the comparison expressed by (16) supersedes that previously given. In particular it is to be noted that the evidence of Tables III, IV, and V indicates that the scales of both the PDP and the PD are sensibly parallel to the Mount Wilson photovisual scale.

Equations (15) and (16) follow from the combination of several color equations and zero-point differences. This indirectness of method implies an accumulation of uncertainty which is not lessened by the qualifications affecting some of the numerical data. The general correctness of (15) and (16) is indicated, however, by comparing their difference, namely,

$$\text{PDP} - \text{PD} = -0.09 + 0.12\,C\,, \qquad (17)$$

with equation (5), which was obtained directly from the 112 stars appearing in both catalogues. This comparison is essentially the same as that described in connection with equation (7), and the same remarks apply here as there.

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[1] *Mt. Wilson Contr.*, No. 288; *Astrophysical Journal*, **61**, 284, 1925. See eighth column of Table VI.

[2] See preceding footnote.

REVIEWS

Thermodynamik der Himmelskörper. By R. EMDEN. A reprint of an article from *Encyklopädie der Mathematischen Wissenschaften*, **6**, 2, 24, pp. 373–532. Leipzig: B. G. Teubner, 1926. Pp. 160.

The editors of the *Encyklopädie*, recognizing the activity in recent years relative to problems which lie in the domain of the thermodynamics of the stars, intrusted the preparation of an article reviewing this literature to Professor R. Emden, and a better choice of an author could scarcely have been made. The extent of this literature during the last fifteen years has been very great, and much of it was of a temporary or tentative character, and therefore obsolete at the present time. It was highly desirable, therefore, to have it reviewed and weighed by Professor Emden himself, for it was Emden's theory of polytropes that furnished the foundations on which more recent developments rested.

The first part of the article, approximately one-half, is devoted to the ideas and results which were contained in Emden's *Gaskugeln* published in 1907. In this part of the work only thermal and gravitational energies are taken into consideration. The modifications of the theory made necessary by the inclusion of the energies of radiation are reviewed in the second half of the article. This part is devoted largely to the work of other authors, particularly that of Eddington, who is responsible for the theory of radiative equilibrium.

From the hydrodynamical equations, which, of course, assume mathematical continuity in the fluid, it is found that any gas sphere which is homogeneous in concentric layers is in equilibrium provided temperature of each layer is properly determined. In a perfect gas the three variables —pressure, volume, and temperature—are related by the equation

$$pv=\frac{R}{m}T\,,$$

where R is the gas constant and m is the molecular weight of the gas. Thus at every point in a pv-plane a temperature is uniquely defined. Any reversible path in this plane along which the heat capacity is constant is a polytrope. Their equations have the form $pv^k=\text{Const.}$, and the exponent k may have any real value whatever.

According to Emden, polytropes were introduced and discussed by

G. Zeuner in 1872, and first applied to cosmic physics by A. Ritter a few years later. In his *Gaskugeln*, Emden chose the polytrope gas spheres for particular and minute examination. A polytrope gas sphere has the property that "if a small portion of the gas (a gas particle) is moved along a radius it will follow the changes of state of a polytrope, and will everywhere and always agree with the displaced particle with respect to pressure, temperature and density." He justified his choice of this particular class of gas spheres from among the totality of gas spheres in equilibrium by the fact that he was able to work out all the details of the structure of such a sphere for any given class of polytrope. Central pressure and central temperature, central density, size and mass, could all be computed when suitable data were given. As an example he constructed a gas sphere of the mass and mean density of the sun with atmospheric air as the substance and found that the central temperature was 456,000,000° C., the central pressure was 42,000,000 atmospheres, and the central density was 33 gm per cubic centimeter. If the substance was hydrogen instead of air, everything else the same, the central temperature was 31,000,000°, while the central density and pressure was the same as before. Thus was introduced the high temperatures with which astronomers everywhere have been made familiar in more recent years.

One of the great virtues of Emden's book was that he did not take his theory too seriously. With respect to the foregoing figures he remarked, "These values are so great that they lie entirely outside the range of experiment. But the investigation loses nothing of value if these magnitudes are regarded merely as symbols of calculation." He did not regard the theory as complete and was very cautious in his applications to the stars. He was dealing with gas spheres, not stars—and the distinction was clear.

Emden's theory of polytrope gas spheres is attractive even though imperfect and incomplete. It is an obvious blemish—in so far as its application to stars is concerned—that polytrope gas spheres have a zero density and a zero temperature at the surface. The amount of matter near the surface is small, and he would abandon the concept of continuity of the fluid near the surface in favor of a treatment by the kinetic-gas theory. This may be necessary from the point of view of a physicist, but it spoils the beauty of the theory from the point of view of a mathematician.

There is another blemish in the theory which is far more serious, namely, there is no proof that any spherical mass of gas would actually have the structure of a polytrope; in other words, there is no proof that polytrope gas spheres are stable structures, and without such a proof the theory remains incomplete at a vital point.

On page 20 of his *Gaskugeln* he concludes: "*If a* [polytrope] *gas mass is in equilibrium, and if in its exchanges of heat it can only follow a cosmogonide, then for* $\gamma > \frac{4}{3}$ *it is stable, and for* $\gamma < \frac{4}{3}$ *it is unstable.*" The word "polytrope" is inserted by the reviewer, but the propriety of doing so is obvious.

At the close of chapter vii, page 118, he draws the conclusion: "*If a polytrope gas sphere changes its radius in such a way as to remain always a polytrope gas sphere of the same class, every particle of it moves along a cosmogonide.*"

This is a strong hypothesis, but combining these two theorems in chapter viii, page 121, in which he is discussing the energetics of the contraction theory (it will be remembered that in 1907 the contraction hypothesis of Helmholtz was the dominant hypothesis as to the origin of the sun's heat), he derived the "fundamental theorem": *In order that their equilibrium may be stable or neutral, polytrope gas spheres of an arbitrary class k must be of 1-, 2-, or 3- atomed gases. Unstable equilibrium exists for multiple atomed gases. Adiabatic gas spheres exist in stable or neutral equilibrium only in the three classes* $\gamma = \frac{5}{3}$, $\gamma = \frac{7}{6}$, *and* $\gamma = \frac{4}{3}$. *Polytrope gas spheres for an arbitrary k can be constructed only for those kind of gases which give stable or neutral equilibrium for adiabatic spheres.* One sees from this that Emden's stability theorem holds only for polytrope gas spheres that are constrained to remain polytrope gas spheres of the same class. It does not assert that a free polytrope gas sphere is ever a stable structure.

The reviewer has only words of praise and admiration for the excellent and thorough work exhibited by Emden in his *Gaskugeln*. His attitude toward his own work was modest and thoroughly sincere. He entertained no illusions as to its limitations or lack of completeness. He did, apparently, entertain grave doubts as to the applicability of the theory to actual stars; rather stronger doubts then than are expressed in the present article. A kind of growth in faith takes place with the lapse of time. When Simon Newcomb determined the rate of advance of the perihelion of Mercury he found his figures discordant. He published his result 43″ per century with much misgivings. A year later he referred to it as probably correct, and many years later all doubt had vanished. Sir George Darwin had a similar experience with his theory of tidal friction.

In 1916 Sir Arthur Eddington made a noteworthy contribution to the *Thermodynamics of Gas Spheres* by adapting and extending Schwarzschild's theory of radiative equilibrium for plane layers to radiative equilibrium of spherical concentric layers. The idea of radiation pressure was an old one, and R. A. Sampson had suggested its importance in the equi-

librium of the stars some twenty years before, but it was formulated for gas spheres in 1916. The effect of this addition to the theory was to relieve somewhat the high densities and pressures at the center, the temperatures, however, still remaining high. It was found that the radiation within the star, under certain assumptions, acts like a polytrope gas sphere for which the polytrope class number is 3. Thus the polytrope character of the gas sphere theory was preserved, and an especial importance given to the class $n=3$.

In 1915 the reviewer,[1] who was interested in the origin of stellar energies, devised the hypothesis that the stellar energies were derived from a complete consumption of substance of the star ("annihilation," to use the picturesque word subsequently applied to it by Jeans, although annihilation, in the sense of making something become nothing, is not altogether correct) and that a rematerialization of energy took place in the cold and vast stretches of astronomical space. It followed naturally from this hypothesis that the densities and the luminosities of the stars were functions of the stellar masses, the luminosities increasing and the densities decreasing as the masses increase.

This hypothesis acted as a natural barrier to the acceptance of a gas-sphere theory for the structure of the stars. As is well known, when a gas is near the critical stage of pressure and temperature at which condensation occurs, the gas laws fail, that is, the state of the gas is no longer predicted by them. According to this hypothesis, another critical point occurs in the state of a gas when the pressure and temperatures are of stellar dimensions. The very existence of the gas itself is threatened. Can anyone suppose that the gas laws continue to hold, quite unaffected by this new state of things? Would they not rather warn us, if we could but read them properly, that an extraordinary change was impending? It would certainly seem so.

Like all astronomers and physicists in 1916, Eddington regarded the atom as a permanent physical unit. He had nothing to warn him that he was dealing with a critical state, and he was quite ready to extend the gas laws from the laboratory realm of experience to the stellar realm of inexperience. Indeed, he was very optimistic, for everyone knows the danger of extrapolation of a physical law outside of the domain within which it is known to be valid; and the extrapolation in this case was enormous.

In a paper on "The Relations between the Masses and Luminosities of the Stars" in 1924 Eddington plotted the luminosities of some forty

[1] *Astrophysical Journal*, **48**, 40, 1918.

stars against their masses, which, by that time, were fairly well determined, and he found precisely that type of relationship which was anticipated by the reviewer some years before. Aside from a constant factor, however, the gas theory gave a formula which, when fitted to Capella, gave an excellent representation of the curve on which the stars lay. This notable achievement has since become known as the mass-luminosity law. Its implications, however, contradicted the ideas which Eddington has previously entertained as to the course of stellar evolution, namely, that in its early stages a star was a giant and of vast dimensions, and in its later dwarf stages small in bulk and in luminosity, but constant in mass throughout. On this very slender line of evidence, he, too, felt compelled to assume that the stars consume their own mass, although it would seem that he still regards a star as having condensed from a nebula, on no evidence at all. His argument seemed to be convincing also to Jeans, who adopted the same hypothesis a few months later. Indeed, Jeans puts forward the claim that he invented the idea in 1904, although his claim is hardly justified by the reference which he cites, as he was not considering stellar energies at that time.

With the acquisition of Jeans and Eddington, the hypothesis that the stars consume their own masses soon became current among astronomers, although there are physicists who still listen skeptically, among whom may be mentioned the distinguished relativist, H. Weyl. It would seem as though these physicists do not meditate deeply upon the evidences of geology and astronomy. Even the vast amount of energy released in the destruction of matter is none too great to account for the floods of energy that have been radiated from the stars over the inconceivably long stretches of time which are demanded by the dynamics of the galaxy. It seems to the reviewer that even Jeans and Eddington do not show a full appreciation of it, for a few score of thousands of billions of years (the life-period of a star, according to them) is not sufficient to account for the progress toward a steady state which, as the stellar statistics reveal, has already occurred.

In a general way we are in agreement as to the origin of a planetary system. It is due to the close approach of a visiting star. Both of these distinguished writers regard the solar family of planets as an almost unique structure in the universe, because the life-period of a star (of the order of 10^{13} years) is too small when compared with 10^{15} or 10^{16} years, which is the order of magnitude of the expectation of any given star for a close approach, for many such incidents to have occurred.

The trouble seems to lie in their adoption of the second law of ther-

modynamics as a postulate of cosmology. There is no objection to this adoption if it is done as a matter of free choice, but it is certainly a mistake if they regard it as a matter of compulsion. Physicists are rather inclined to be dogmatic about what "must be," but if they know everything already, further research is unnecessary and a waste of time. The fact is that we know only those processes of nature that have come under our observation, and we cannot safely dogmatize about those that we have not observed. Physical laboratories, even in the aggregate, are but a minute speck in the universe at large, and the processes which we actually observe run through their cycle in a few moments, or at the most in a few years. There is nothing in the experience of the physicists which will justify them in making any assertions as to what will take place in intergalactic space in an interval of time that is to be measured by aeons (10^{15} years). An astronomer, however, has some right to point to the galaxies and to suggest that "this is what has happened"; and if it has happened once, it may happen again—indeed, it is possibly happening continuously. What we see, and all that we have seen, is but the impression of a momentary glance. Is the physicist in a position to say that his knowledge of it is profound? or that in the course of something like 10^{13} years it will all have evaporated and disappeared?

According to Jeans and Edington, the end is fixed and certain. The second law of thermodynamics holds even for those processes of nature about which they know nothing, about which they have not yet even dreamed, and for which the second law cannot even be defined. In a recent novel Talbot Mundy has remarked, "People do not like problems. They want answers, and, generally, they want those answers wrong."

In cosmology the second law of thermodynamics is a postulate, just as the first law, the conservation of energy, is a postulate, to be adopted or rejected according to one's taste. Most people like the doctrine of the conservation of energy, and, in its applications to cosmology, they dislike the second law; but everyone is free to make his own choice. A rejection of the second law of thermodynamics as a postulate of cosmology no more implies a rejection of the doctrine of entropy in physics than a rejection of the doctrine of the conservation of matter as a postulate of cosmology invalidates the assumption of the chemists that mass is constant in the transformations of chemistry.

At the time when the reviewer made the hypothesis that the birthplace of the atoms is in the depths of astronomical space, the only direct evidence that could be pointed out was the blackness of the night skies. In the years that have elapsed since then a second type of direct evidence

has been found in the highly penetrating radiation that comes to us from all directions alike, a radiation that is certainly not connected with the stars or with the conspicuous nebulae. If the protons and electrons result from an absorption of the radiant energy of space, this highly penetrating radiation suggests the building-up of the lighter atoms from these newly born protons and electrons. This is not an improbable result if a number of protons and electrons are produced at the same time and in the same place. Inasmuch as this cosmic radiation contains about 10 per cent of the total energy of starlight, it would seem that about 10 per cent of the energy absorbed in the production of electrons and protons was almost immediately re-radiated in highly penetrating waves.

Does the second law of thermodynamics forbid such a process? If the second law is regarded as a postulate of cosmology, the answer is perhaps in the affirmative. But if the second law is merely a law of physics, the answer is distinctly "No," for the physicists know no more about such a process than the astronomers do.

Jeans and Eddington made a long step forward in cosmology in rejecting the old postulate of the conservation of matter. There remains before them the much less difficult one of rejecting also the second law of thermodynamics as a postulate of cosmology.

One more point that is germane to the present discussion is the origin of the binary stars. According to the current notions, binary stars are the result of a process of fusion due to rotational instability, excessive rotation having been induced by the process of contraction. With the abandonment of the contraction hypothesis of Helmholtz, however, the foundations of the fission theory are removed, and the theory itself rests upon nothing. It is much the same kind of situation as would exist if the mathematicians should still maintain a corollary of a theorem after the theorem itself had been disproved. The fission theory, apparently, is dead, for the life has all gone out of it. Yet more than a third of the stars are binary!

There remains still, however, the hypothesis that a binary or multiple star represents the old age, or final stage, of a planetary system, one of the planets having succeeded in winning its way by growth to starhood. But if this hypothesis is correct, planetary families must be common, in fact rather ordinary, and the uniqueness of our own family disappears. This implies, however, periods of time in the galaxy that are much greater than the 10^{13} years that have been recently assigned to it, and the stars too must be much older; and if the stars are much older, the masses of the stars must somehow be replenished. This would be accomplished by the

existence of nebulosity and solid fragments in space. In order that the mass of the sun should be maintained without loss, the mean density of the space through which it travels must be of the order 10^{-19} gm per cubic centimeter. At the present time astronomers recognize a nebular density of the order 10^{-24}. This is far too small a density to maintain the sun's mass, but there still remains the possibility of nebulosity which is not yet recognized, and also the possibility of fragmental material. The sun certainly cannot maintain indefinitely its present rate of radiation, if 10^{-24} actually represents the density of interstellar space. The reviewer awaits further evidence calmly. Fifteen years ago the majority of astronomers regarded space as altogether empty save for the visible nebulae and the stars.

During the latter half of the nineteenth century Lord Kelvin insisted that the contraction of the stars furnished the only possible source of the stellar energies, notwithstanding the fact that the implications of this hypothesis ran counter to the direct evidences of geology and biology. Notwithstanding the fact that Lord Kelvin dominated the world of physics, the geologists and biologists went their way, accumulating additional evidence, until it grew more and more certain that Lord Kelvin was wrong. He built up a theory of the age of the earth and the distribution of the heat in its interior, in accordance with the evidence which he recognized and the postulates which he chose, that had as little to do with the facts, as we see them today, as the Emden-Eddington theory may have to do with the facts of the internal structures of the stars. One would suppose that Lord Kelvin's experience with the theory of the condition of the interior of the earth would have served as a warning to those who strive to tell us about that much more difficult problem, the internal structure of the stars, but apparently it has not.

The Emden-Eddington theory of gas spheres in radiative equilibrium is a very interesting and valuable discipline, but the reviewer questions its applicability to the structure of the stars for the following reasons:

First.—There is no adequate reason for supposing that the substance of a star is a gas in the sense that it everywhere obeys the gas laws. Indeed, a large point of view would lead us to suppose that it is not a gas.

Second.—When the sources of the stellar energies are taken into consideration, there is no evidence that an equilibrium theory of any kind is applicable. The phenomena of sun-spots and stellar variability do not suggest dynamical equilibrium.

Third.—Even though the first two objections be waived, there is no

evidence that any actual star has a polytrope structure. If not, the theory again is not applicable.

Fourth.—The theory is abandoned, even by its authors, as not applicable at the stellar surface.

At the conclusion of his masterly paper on "The Figures of Equilibrium of Rotating Fluid Masses," Poincaré made a remark to the effect that "one may be tempted to use the present results [theory of fission] to prove or disprove the Laplacian hypothesis, but *one must not forget that the conditions may be very different.*"

W. D. MacMillan

University of Chicago
March 1931

Critique of Physics. By L. L. Whyte. New York: W. W. Norton Co., 1931. Pp. xi+196. $3.00.

The history of modern science has certainly shown that searching criticism of its fundamental processes is an effective weapon of the physicist, if well timed and followed up by constructive activity. The author of the present book believes it possible to clear the ground for a comprehensive "unitary theory," i.e., a fusion of electromagnetism and gravitation, including quantum phenomena, only by such destructive criticism. As he points out, both relativity and quantum mechanics grew out of a realization of definite restrictions on the classical space-time frame of reference; accordingly, Mr. Whyte's thesis is that for further progress still more drastic revision is necessary. This is to consist in the entire elimination of extended space-time co-ordinates, at least from the fundamental axioms, leaving only the ordering and causal connections of space-time coincidences.

Unfortunately, the book hardly progresses beyond the destructive-critical stage, the constructive-active part being confined to a lengthy discussion of the general *type* of new theory required and formulation of a program for future research. Nevertheless, it is, in places, highly suggestive, and may well prove to have foreshadowed some of the impending developments in quantum theory: for example, the growing realization of the inadequacy of the classical concept of time in relativistic quantum mechanics.

F. C. Hoyt

ERRATA

We greatly regret that by some error the Greek letter eta, correctly given in the first proofs, was changed to epsilon in the title of Dr. C. J. Krieger's paper in the July issue of this *Journal*. This occurred both in the title of the paper and in the Table of Contents on the front cover page. Eta was given correctly in the running heads of the pages.

Accordingly, on the cover title, as well as on page 10, of the *Astrophysical Journal*, Volume 74, *for* epsilon *read* eta, making the correct title, "The Spectrophotometric Study of η Aquilae."

NOTICE TO CONTRIBUTORS

There is occasionally published in the *Astrophysical Journal* a Standing Notice (for instance, on pages 179-180 of the number for September, 1917). This is principally intended to guide contributors regarding the manuscript, illustrations, and reprints. This notice contains the following paragraph:

> Where unusual expense is involved in the publication of an article, on account of length, tabular matter, or illustrations, arrangements are made whereby the expense is shared by the author or by the institution which he represents, according to a uniform system.

The present sheet has been printed for amplifying further that paragraph.

The "uniform system" according to which "arrangements are made" is as follows: The cost of composition in excess of $50, and of stock, presswork, and binding of pages in excess of 40 pages, for any one article shall be paid by the author or by the institution which he represents at the current rates of the University of Chicago Press. When four articles from one institution or author have appeared in any one volume, on which the cost of composition has amounted to $50 each, or when the total cost of composition incurred by the *Astrophysical Journal* on articles for one institution has reached the sum of $200, the entire cost of the composition, stock, presswork, and binding of any additional articles appearing in that volume shall be paid by the author or by the institution which he represents.

As to illustrations, the arrangements cannot be quite as specific, but it may be generally assumed that not more than three halftone inserts can be allowed without payment by the author. The cost of paper, presswork, and binding for each full-page insert is about $8.00, aside from the cost of the halftone itself. In the matter of zinc etchings, considerable latitude has to be allowed, as in many cases diagrams take the place of more expensive tables. It may be assumed, however, that it will seldom be possible for the *Journal* to bear an expense of over $25 for diagrams and text illustrations in any one article.

Contributors should notice that since January, 1917, it has been impossible to supply any free reprints of articles.

Reprints of articles, with or without covers, will be supplied to authors at cost. No reprints can be furnished unless a request for them is received before the *Journal* goes to press.

Every article in the *Astrophysical Journal*, however short, is to be preceded by an abstract prepared by the author and submitted by him with the manuscript. The abstract is intended to serve as an aid to the reader by furnishing an index and brief summary or preliminary survey of the contents of the article; it should also be suitable for reprinting in an abstract journal so as to make a reabstracting of the article unnecessary. For details concerning the preparation of abstracts, see page 176 in the September, 1928, number of the *Journal*.

THE EDITORS

PREPARATION OF ABSTRACTS

Every article in the *Astrophysical Journal*, however short, is to be preceded by an abstract which should be submitted by the author with the manuscript. In order that the abstract may aid the reader by furnishing an index and brief summary of the contents of the article, and may also be suitable for reprinting in an abstract journal, it should be of the type of those which have been appearing in the *Astrophysical Journal* since 1920. It is requested that the abstracts be prepared in accordance with the following:

DIRECTIONS AND RULES

1. *Notes.*—Read the article carefully, taking rough notes covering all the new information reported, keeping a specially sharp lookout for new incidental results and suggestions not directly related to the main subject.

Material not new need not be described; a valuable summary of previous work, however, should be noted.

2. *Subtitles.*—Write, first, a title describing the group of results forming the main contribution of the article, including all that belong together. If there are in addition results which do not come under that title, gather them into as few groups as possible and formulate a complete and precise title for each.

The subtitles should together form a complete index of the new information; that is, they should include every measurement, observation, method, suggestion, and theory which is presented as new and of value in itself. They should be complete in themselves and independent of the main title of the article.

3. *Text.*—Write a paragraph summarizing the main group of results and including the corresponding subtitle either all at the beginning or with parts scattered through the text; and then do likewise for the other groups.

A separate paragraph should be used for each distinct subject involved, but no more than necessary. All material which can easily be grouped together under a single title should be summarized in the same paragraph. Parts of subtitles may be scattered through the text, but the subject of each paragraph must be given at the beginning.

The text should summarize the author's conclusions and should transcribe all numerical results of general interest, including all that might be looked for in a table of astronomical and physical constants, with an indication of the accuracy of each. It should give all the information that anyone not a specialist in the particular field involved might care to have in his notebook.

Complete sentences should be used except in the case of subtitles. The abstract should be made as readable as the necessary brevity will permit. Italicize subtitles but no other words or phrases.

4. *Final checking.*—Re-read the article so as to check the abstract and correct any omissions and mistakes; read the subtitles by themselves to see that they properly index the information; and read the abstract to see whether it cannot be condensed and its English be improved.